Medical Image Registration

Paul Aljabar, Ph.D.

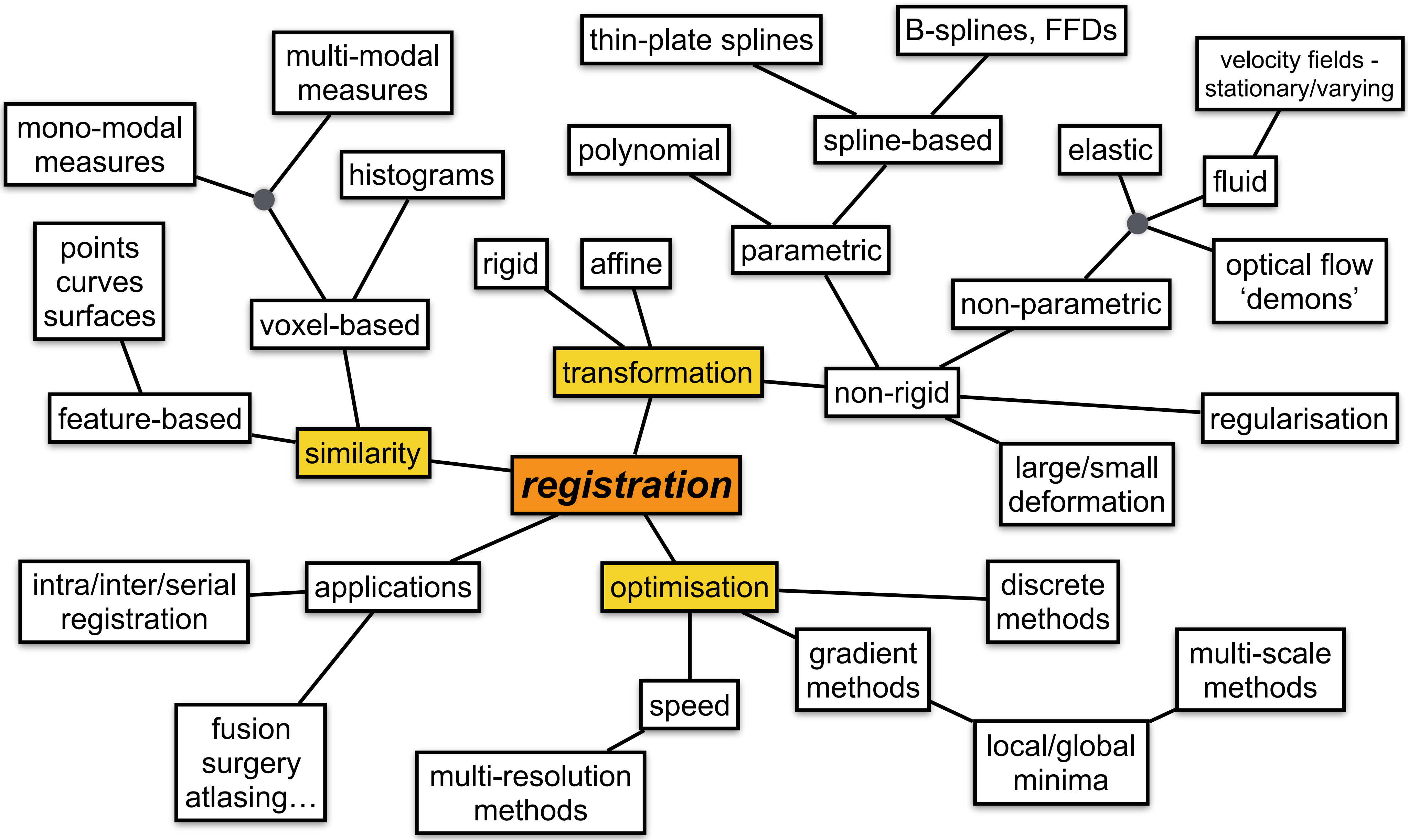
Department of BioMedical Engineering KCL

Adapted from Daniel Rueckert's slides

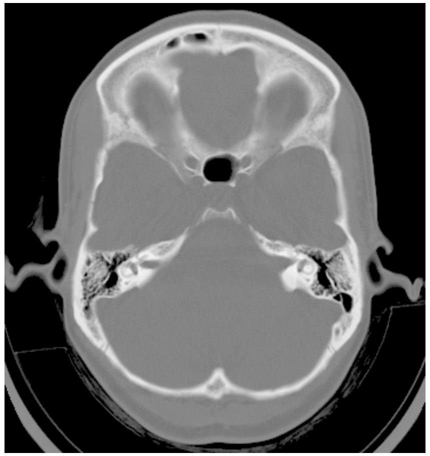
Overview

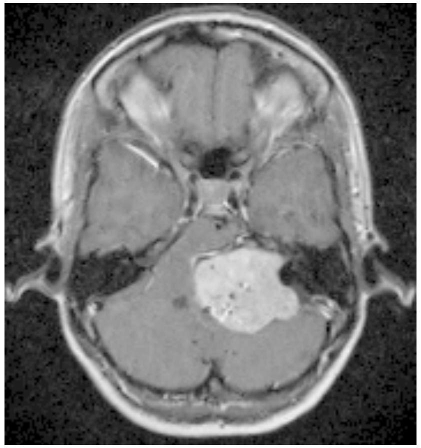
- Introduction to image registration
- A.k.a 'finding a good alignment between images'
  - Transformation models
  - Similarity measures
  - Optimisation techniques (briefly)
  - Rigid and non-rigid registration algorithms
- Examples

Another overview

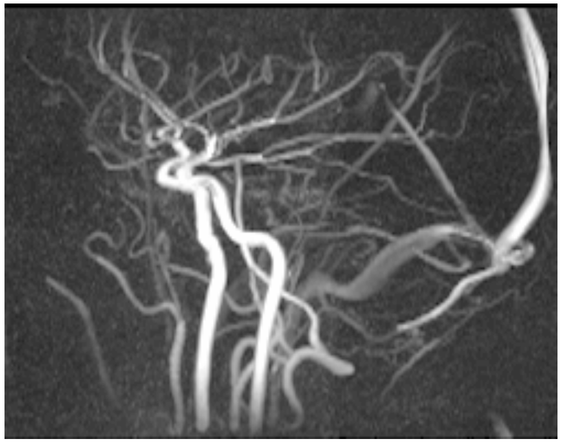


What is image registration?



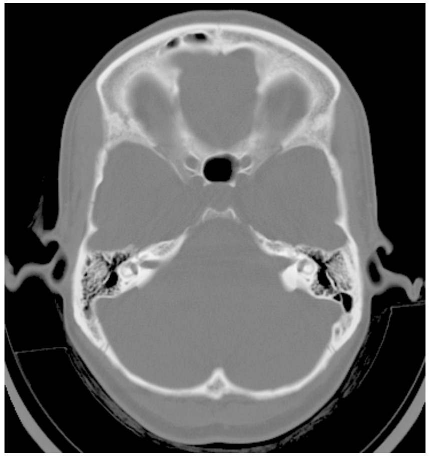


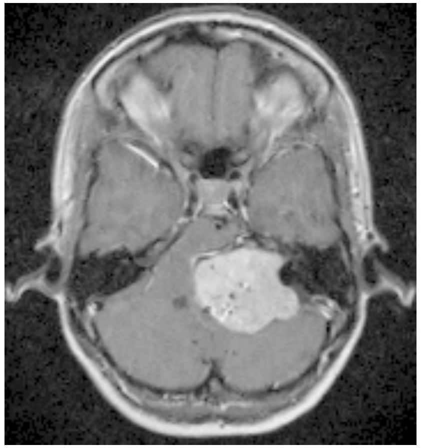
A motivating example

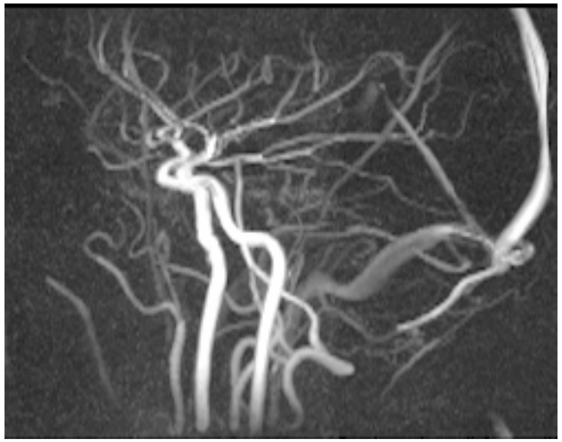


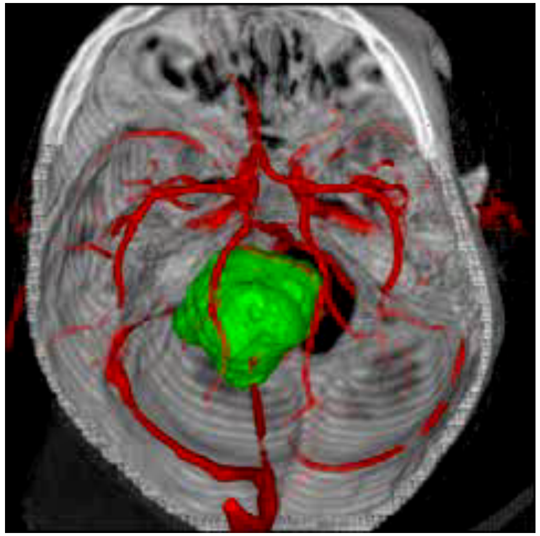
C. Ruff, 1995

What is image registration?









C. Ruff, 1995

Image to image registration

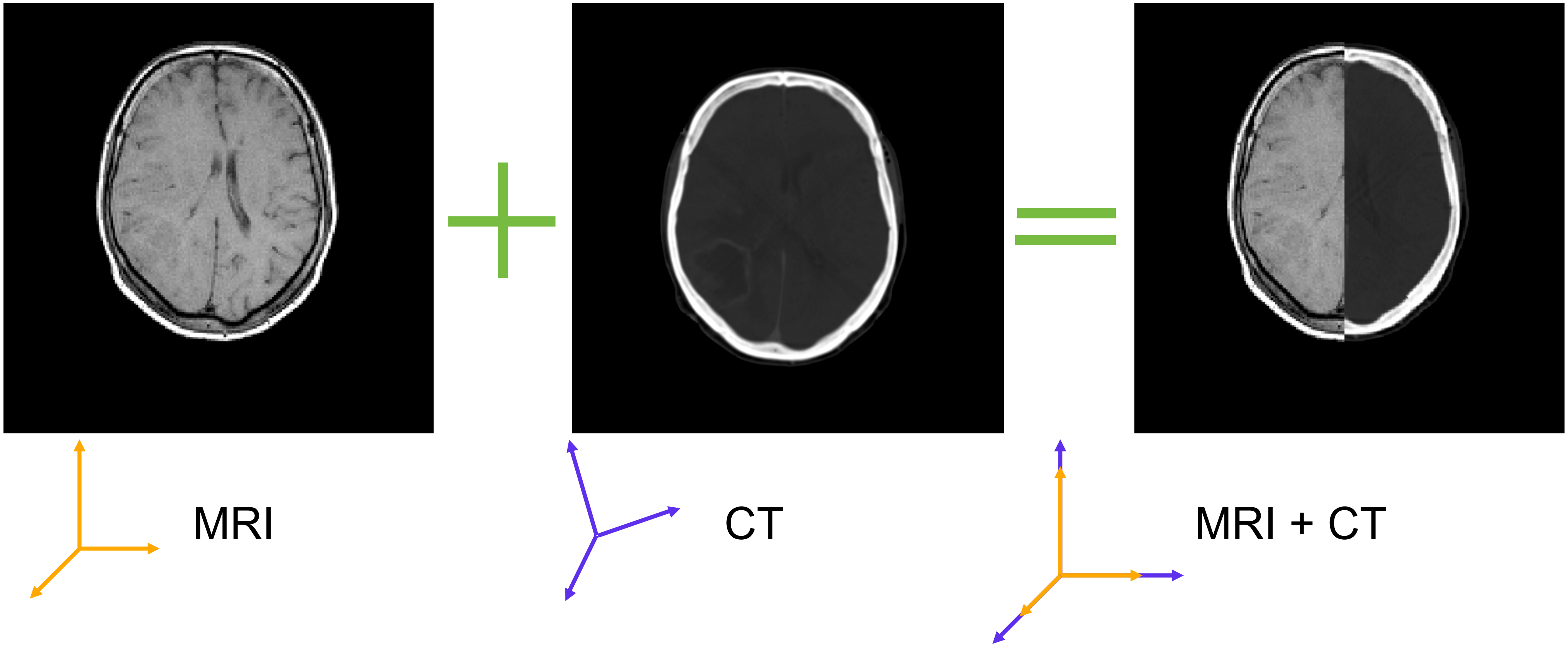
- Intra-subject registration
  - Aim: Registration of different images of the same subject.
  - Purpose: Combine anatomical and functional information from different imaging modalities.
  - Examples:
    - Registration of CT and MR images of the brain for surgery and therapy planning
    - Registration of MR and SPECT/PET images for localisation of tracer uptake to indicate brain physiology or of the head and neck for cancer staging

Image to image registration

- Inter-subject registration
  - Aim: Registration of images of different subjects.
  - Purpose: Assess morphometric variability of anatomical structure across individuals.
- Serial registration
  - Aim: Registration of a sequence of images of the same subject (over time).
  - Purpose: Monitor temporal changes; assessing drug treatment, disease progression in conditions such as epilepsy, multiple sclerosis.

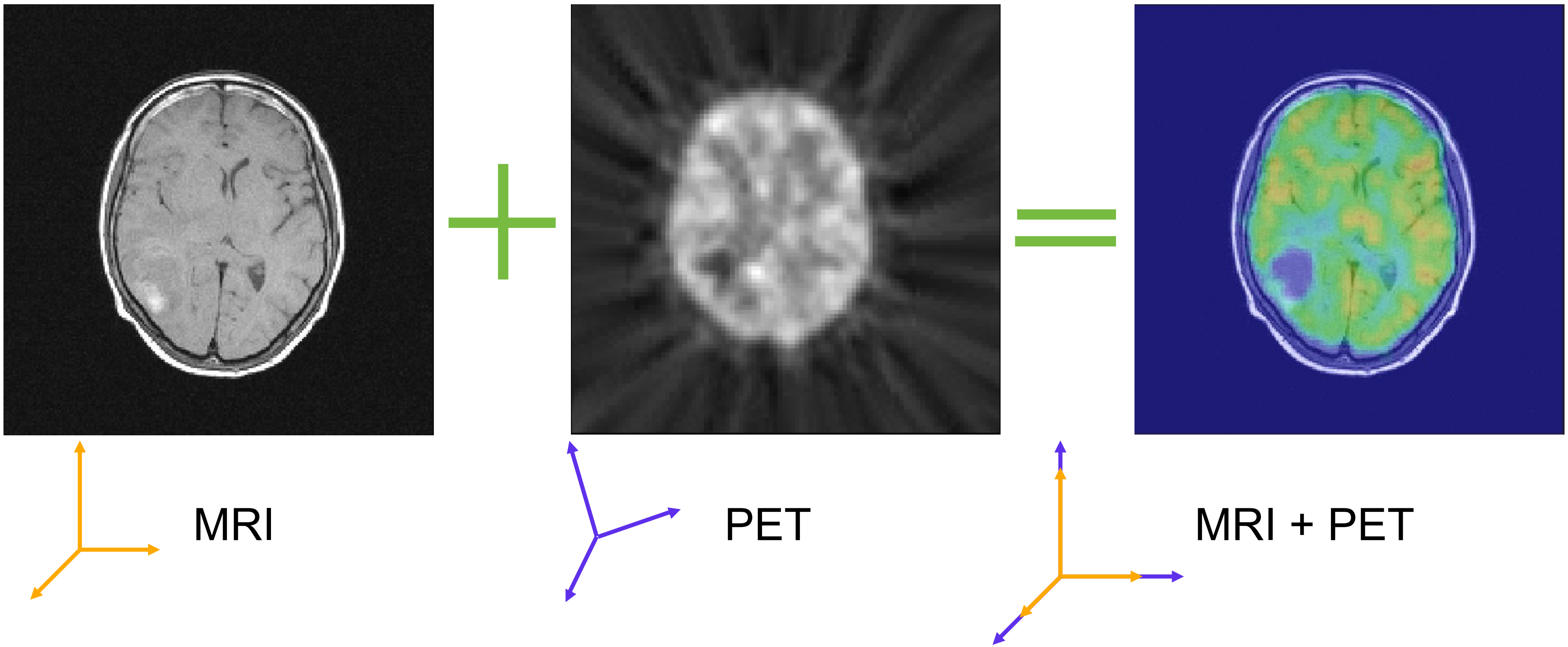
Intra-subject registration example

- Fusion of MR and CT head image:

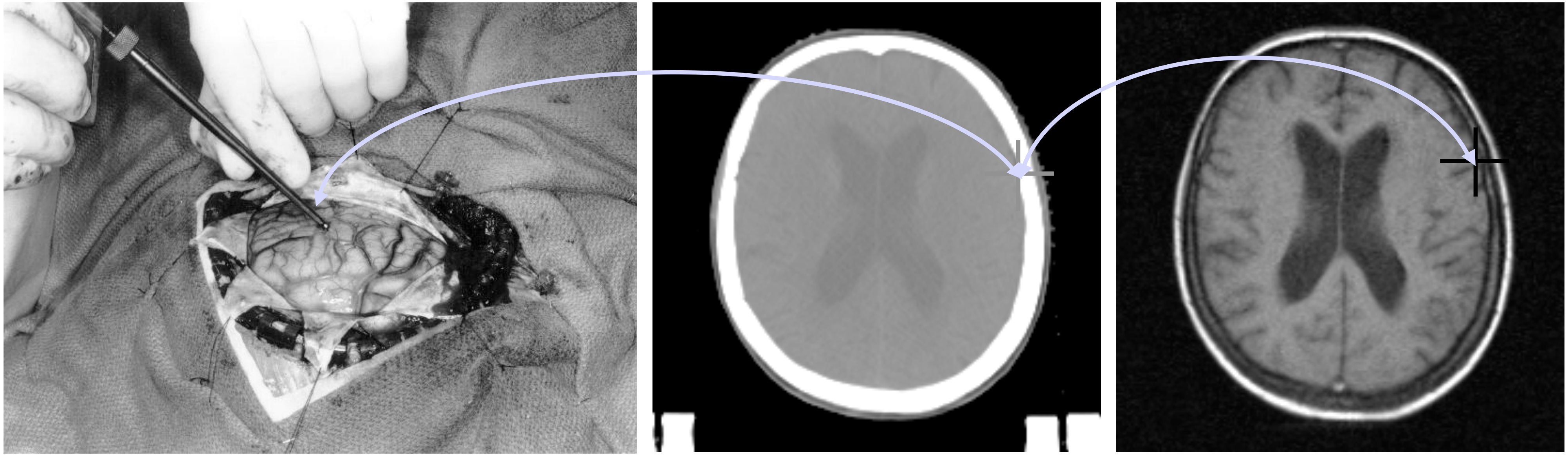


Intra-subject registration example

- Fusion of MR and PET head image :

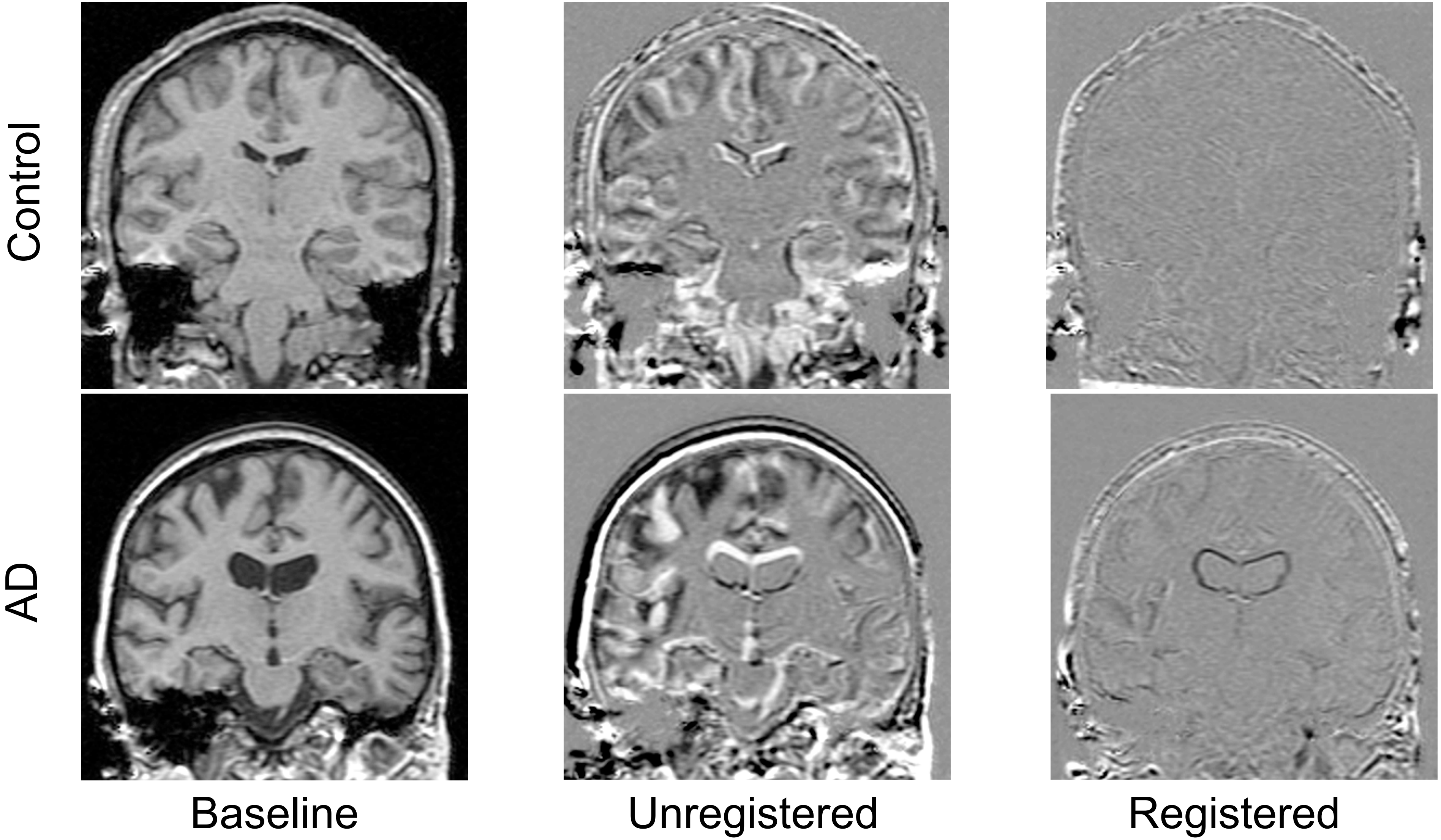


Intra-subject registration example

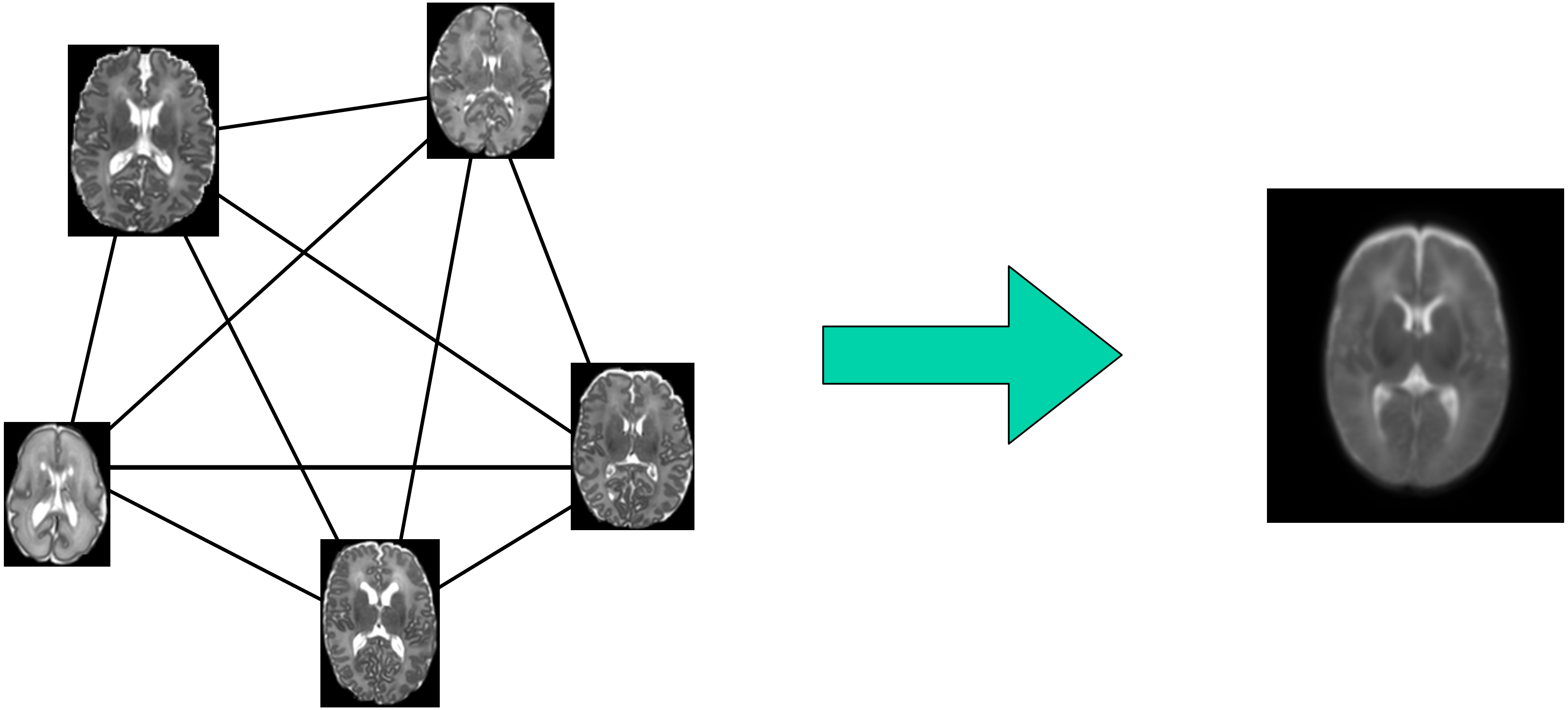


Registration of pre-operative CT and MRI to intra-operative scene

Serial registration example



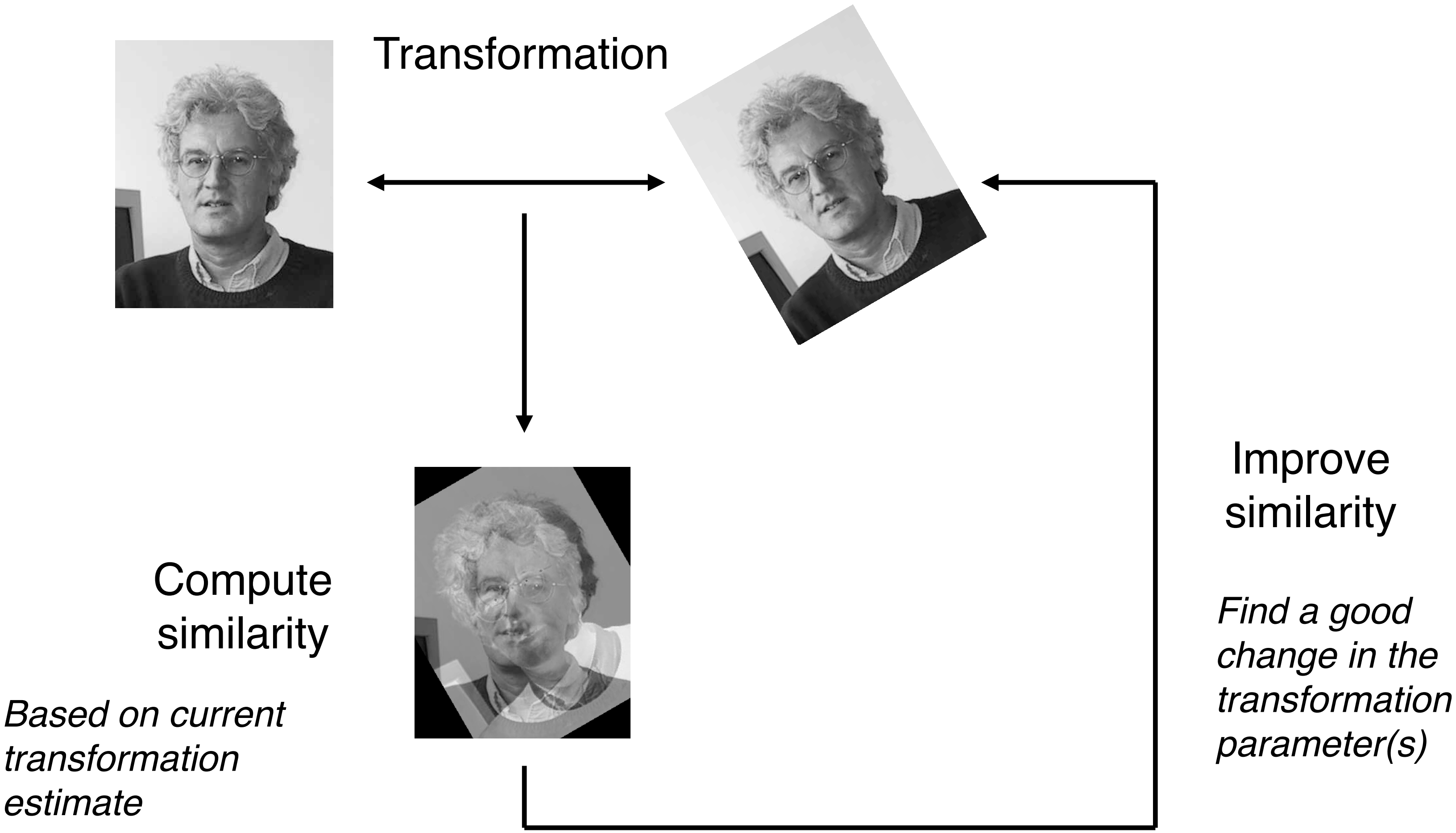
Inter-subject registration example



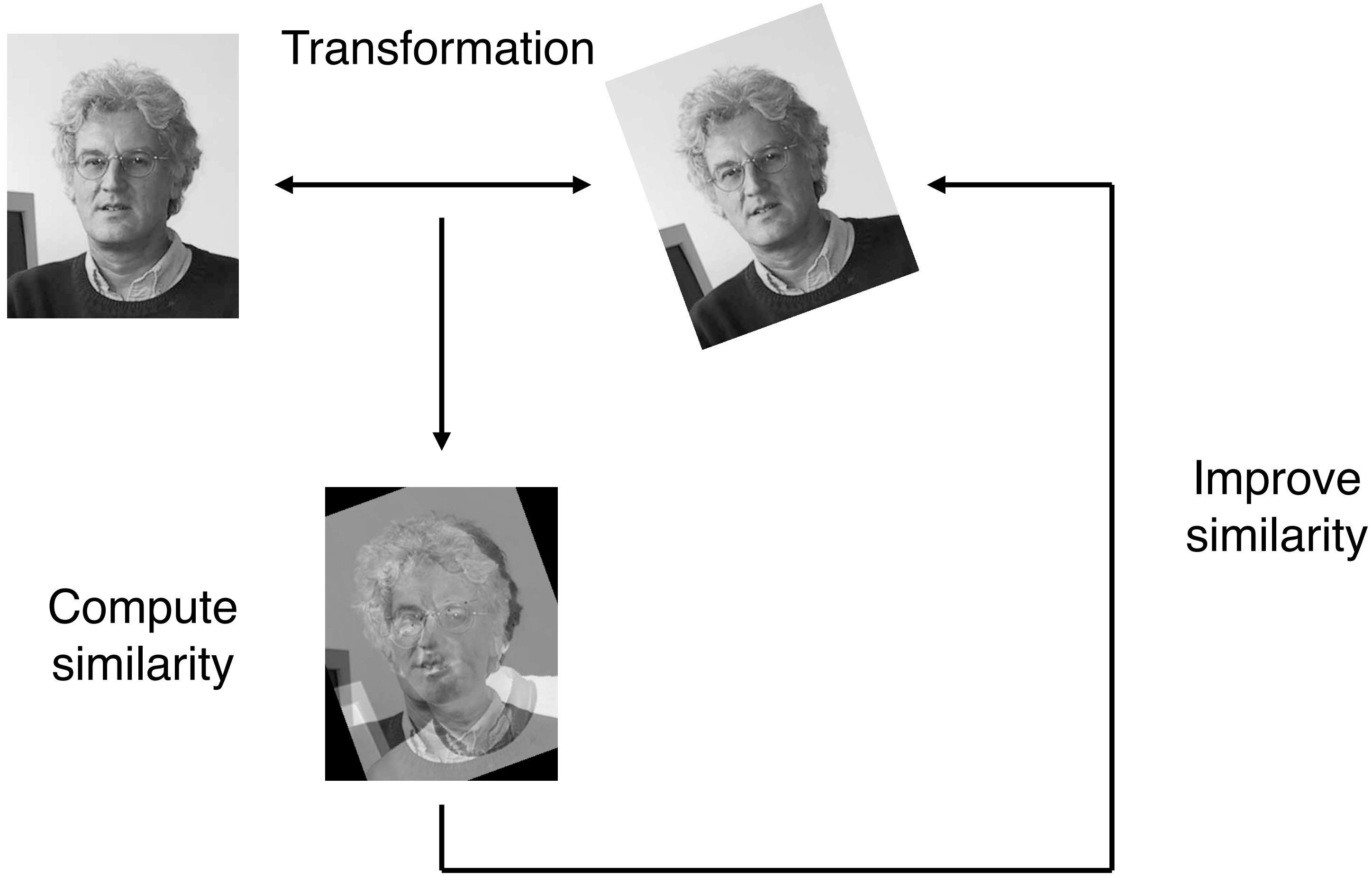
Registration of pairs of images from different subjects
Construction of an atlas (average) representation of anatomy

What is image registration ?

Registration method in a bit more detail



What is image registration ?



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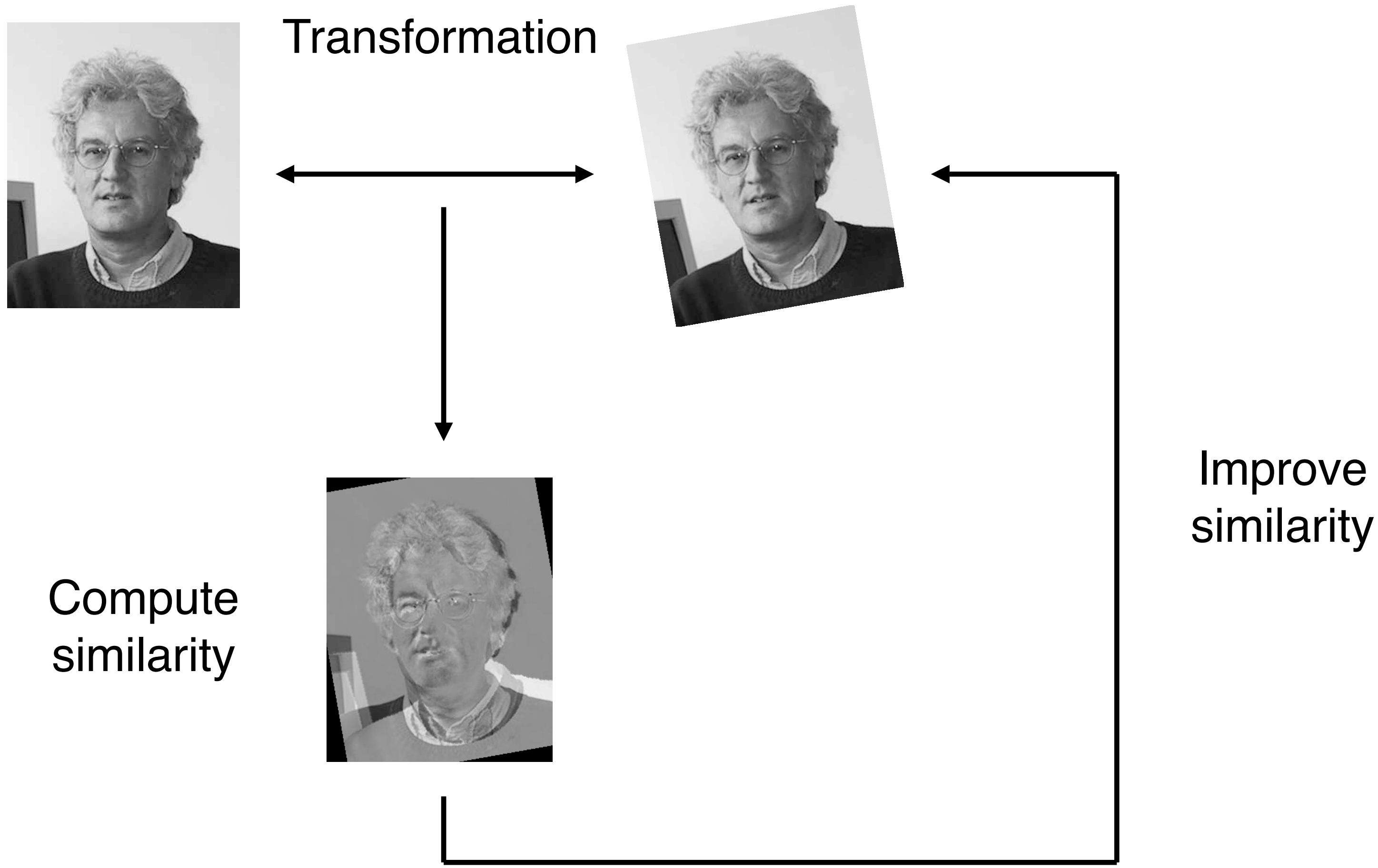

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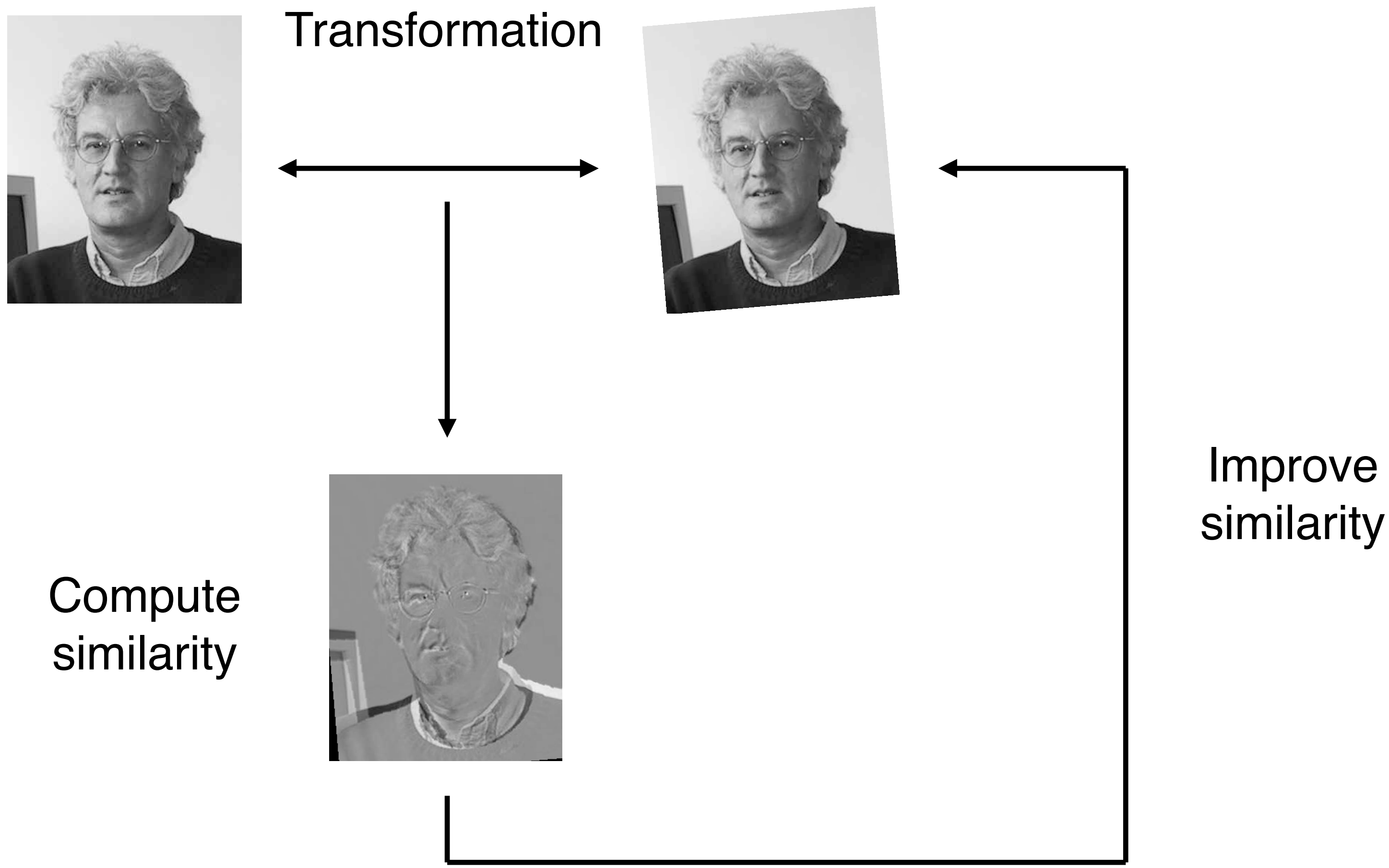


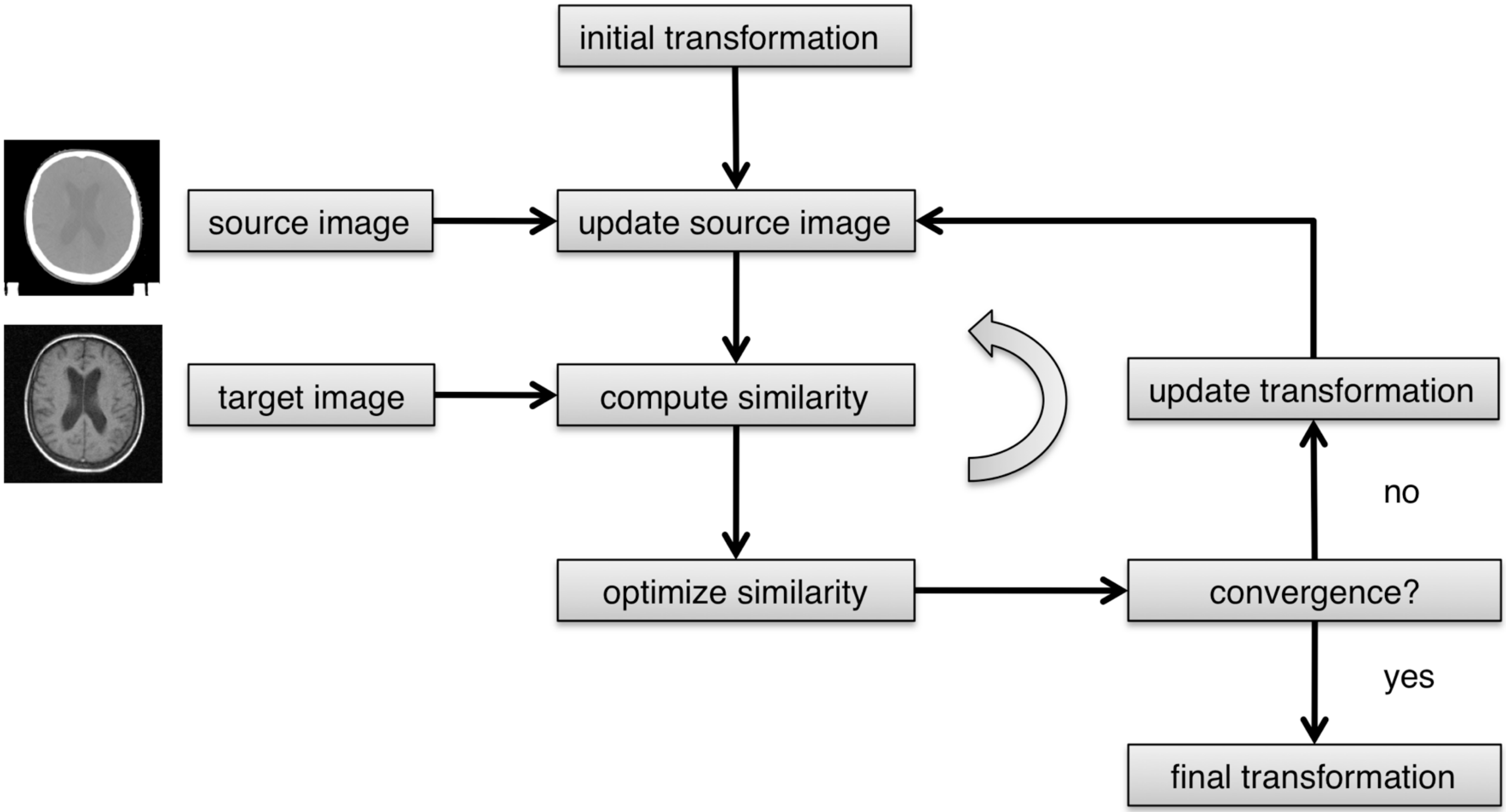
What is image registration ?

Transformation

Compute
similarity

done

Image registration as an optimisation problem



Components of any generic registration algorithm

- Similarity measure
  – How similar are the images/features after registration, in other words how *good* is the registration?

- Transformation model
  – What type of transformations are allowed. Are there constraints that the transformation model should satisfy?

- Optimisation method
  – How do we find the transformation that maximises the similarity? May need to incorporate constraints.

Overview

- Transformation models
  - Rigid
  - Affine
- Similarity measures
  - Mono-modal registration
  - Multi-modal registration

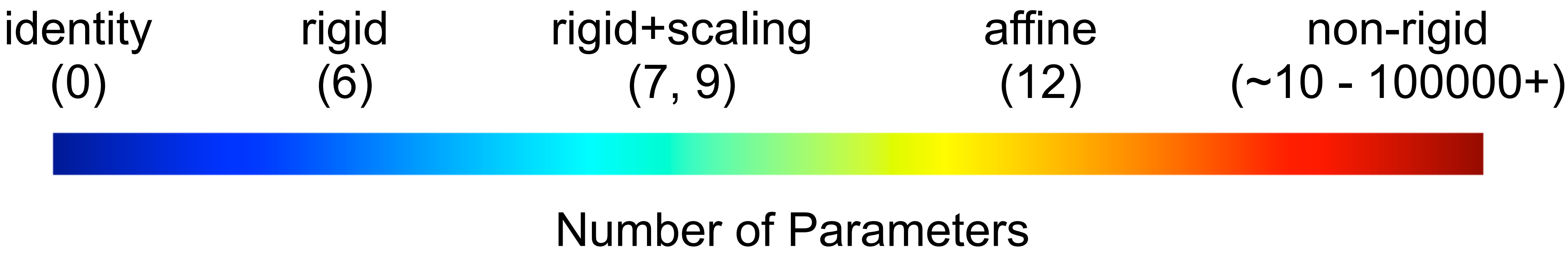
Rigid/affine registration

- Transformation models
  - Splines
  - Elastic and fluid models
  - Regularisation

Non-rigid registration

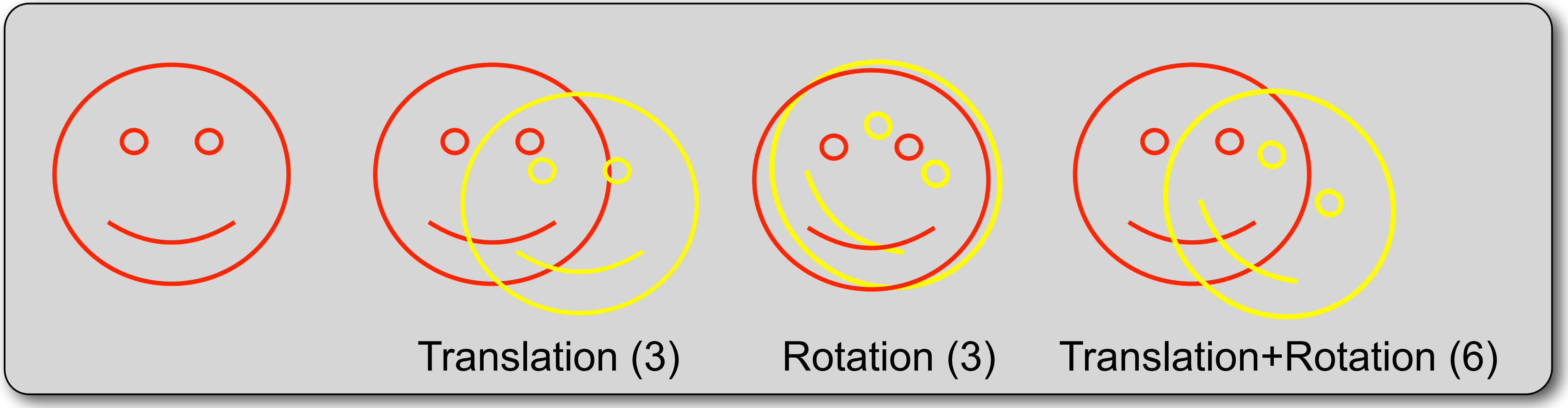
Transformation models

- A transformation can be described by the number of parameters it has
- Often referred to as *Degrees of Freedom* or DOF



Rigid transformation

- Compensates for global patient repositioning
- Preserves distances and angles.



- Appropriate for:
  - brain, bone, optically tracked surgical instruments
  - often used to initialise non-rigid registration

Rigid transformations

- Rigid transformation (6 degrees of freedom)
- Can be written as a single matrix multiplication

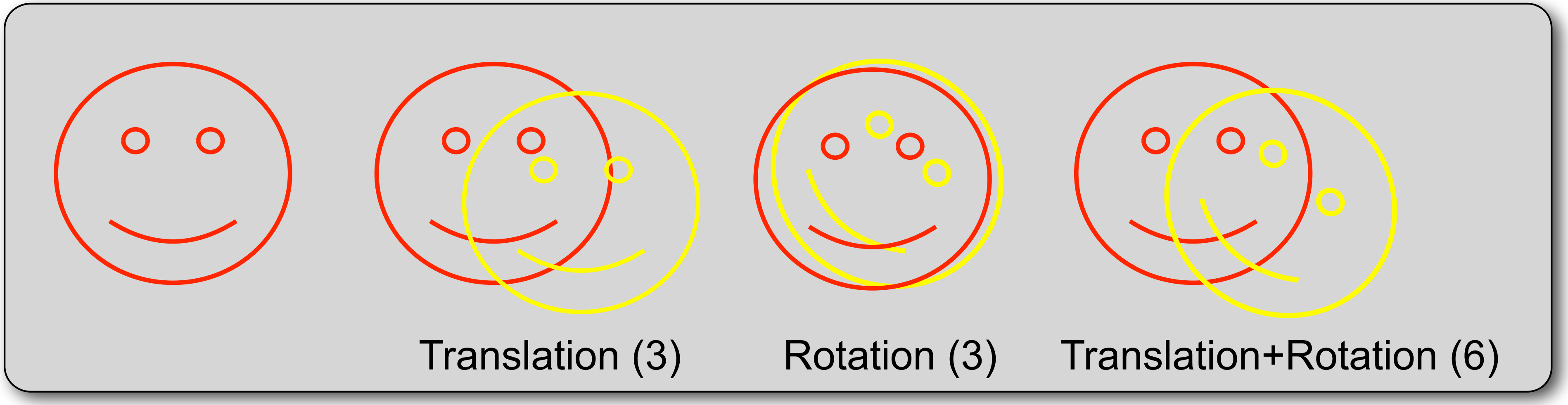
$$\mathbf{T}_{rigid}(x, y, z) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix}$$

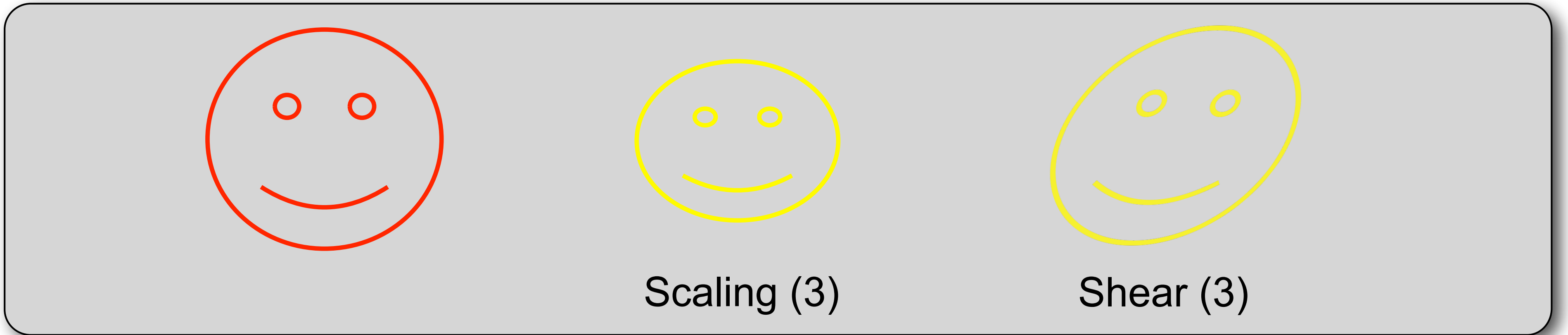
- t_x, t_y, t_z describe 3 translations in directions x, y, z
- $r_{11}, \ldots, r_{33}$ describe the 3 rotations around the axes x, y, z

- Uses *homogeneous coordinate* notation

Affine transformations

- Compensates for additional global scaling and shears





Affine transformations

- Some more matrices

$$\mathbf{T}_{scale} : \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{T}_{shear} : \begin{pmatrix} 1 & s_{xy} & s_{xz} & 0 \\ 0 & 1 & s_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Affine transformations (12 degrees of freedom)

$$\mathbf{T}_{affine}(x, y, z) = \mathbf{T}_{shear} \circ \mathbf{T}_{scale} \circ \mathbf{T}_{rigid} \circ (x, y, z)$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = M_{shear}\, M_{scale}\, M_{rigid} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{12} & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Similarity measure categories

- ***Feature-based*** similarity measures of specific geometric features
  - e.g. points, lines, ridges, surfaces, curvature extrema
  - registration minimises distance between corresponding features
  - correspondence is *interpolated* between features
- ***Voxel-based*** similarity measures of intensities across the whole image
  - registration maximises a measure of image *intensity similarity*
  - correspondences are used everywhere during the registration

Registration based on voxel-based similarity measures

- Registration based on geometrical features
  - Requires points, lines or surfaces to be extracted
  - Registration accuracy affected by localisation errors during the feature extraction
- Registration based on voxel similarity measures
  - Uses some measure directly derived from voxel intensities
  - Assumes there is a relationship between the intensities of both images when they are aligned
  - Does not require any feature extraction, so registration accuracy is not affected by localisation errors

Registration based on voxel-based similarity measures

If a registration uses …

- Geometric features
  - independent of image modalities that give the features (e.g. edges)

- Voxel similarity measures
  - must distinguish between
    - mono-modality registration:
      - CT-CT, MR-MR, PET-PET, etc
    - multi-modality registration
      - MR-CT, MR-PET, CT-PET, etc

Will focus on voxel based registration. For more details of feature based approaches, please see the appendix

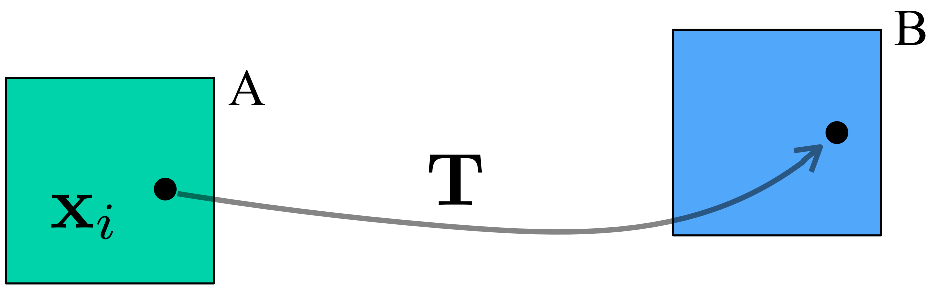
Mono-modal image registration

- Sums of Squared Differences (SSD)

$$S = \frac{1}{N} \sum_{i} \left(I_A(\mathbf{x}_i) - I_B(\mathbf{T}(\mathbf{x}_i)) \right)^2$$

For all voxels i

  - Assumes an identity relationship between image intensities in both images
  - Optimal measure if the difference between both images is Gaussian noise
  - Sensitive to outliers



Mono-modal image registration

- Robust statistics can be used to reduce the influence of outliers on the registration
- Sum of absolute differences (SAD)

$$S = \frac{1}{N} \sum_i | \, I_A(\mathbf{x}_i) \, - \, I_B(\, \mathbf{T}(\mathbf{x}_i) \,) \, |$$

  – Still assumes an identity relationship between image intensities
  – less sensitive to outliers

Mono-modal image registration

- Cross Correlation (CC)

$$S = \frac{\sum \left(I_A(\mathbf{x}) - \mu_A\right)\left(I_B(\mathbf{T}(\mathbf{x})) - \mu_B\right)}{\sqrt{\left(\sum \left(I_A(\mathbf{x}) - \mu_A\right)^2\right)\left(\sum \left(I_B(\mathbf{T}(\mathbf{x})) - \mu_B\right)^2\right)}}$$

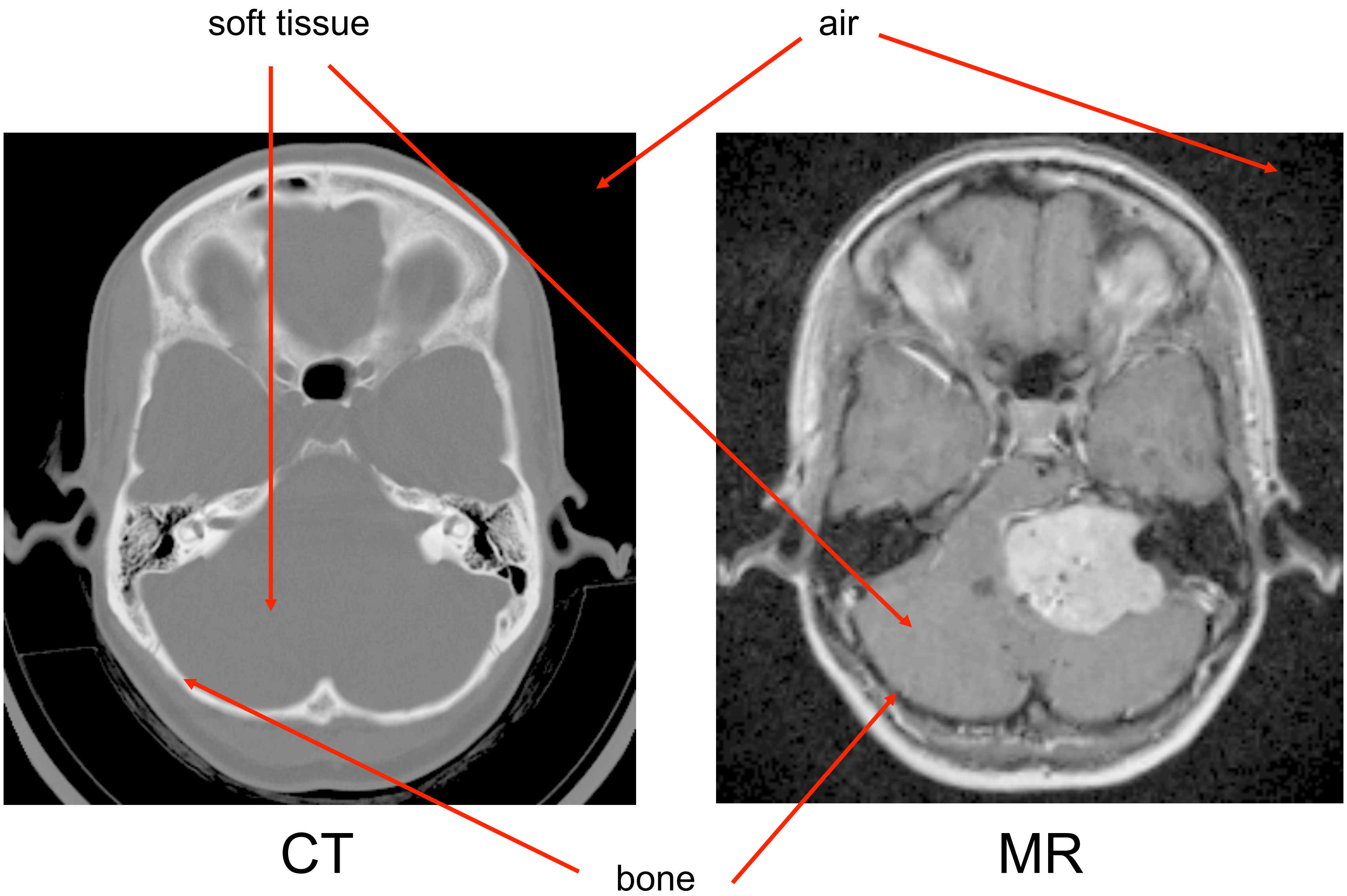
  - μ_A is the average intensity in image A
  - μ_B is the average intensity in image B
  - assumes a linear relationship between image intensities
  - useful, for example, if images have different intensity ranges (common in MR)

Registration: Mono-modal vs Multi-modal

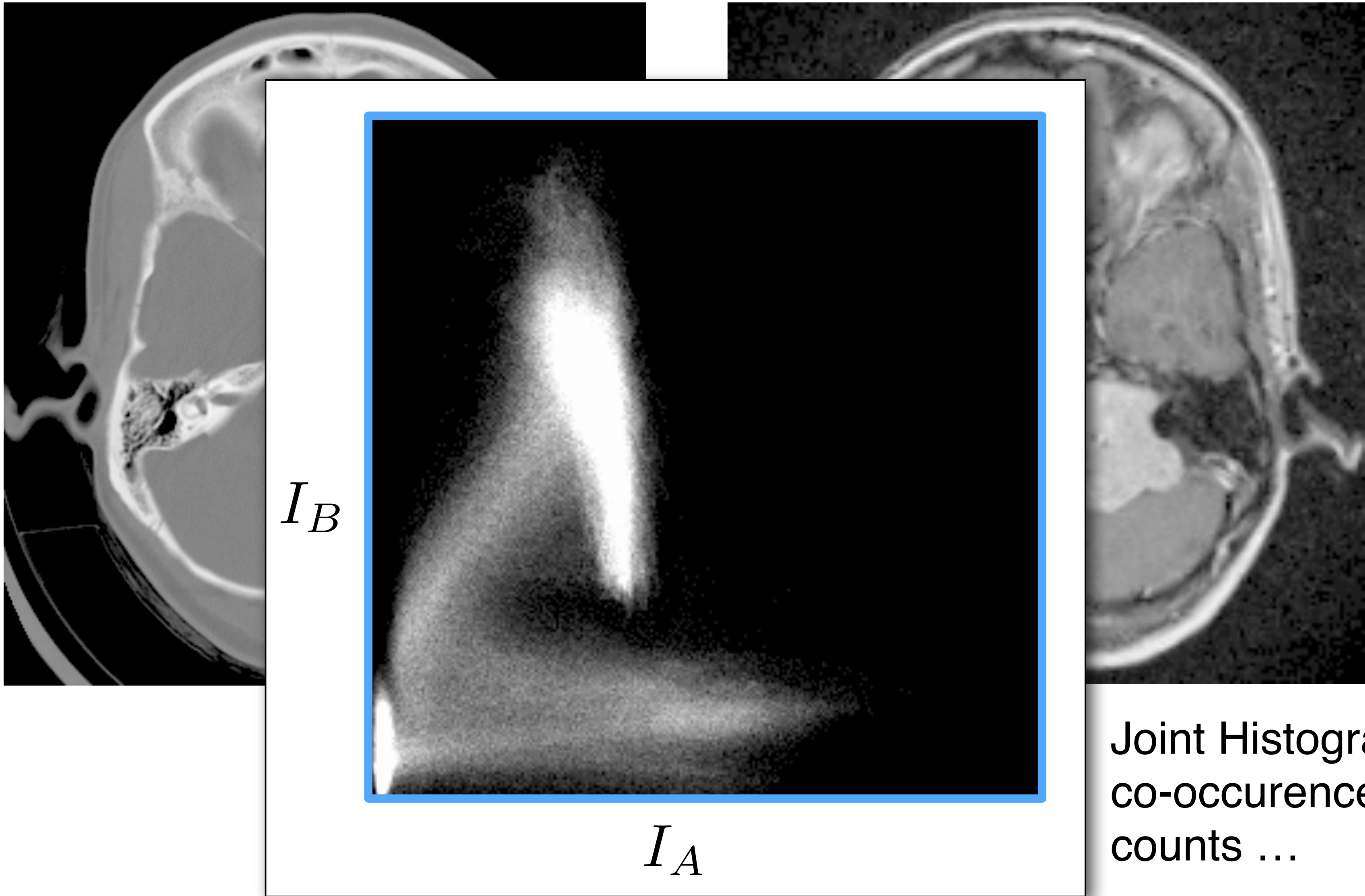
- Mono-modal
  - Intensities are related by some simple function
    - identity: Use SSD (e.g. CT to CT registration)
    - linear: Use CC (e.g. MR T1 to MR T1 registration)

- Multi-modal
  - Intensities related by an unknown function/statistical relationship
  - Relationship between intensities can be viewed by inspecting a 2D *histogram* or *co-occurrence matrix*

Multi-modal images: Measuring similarity

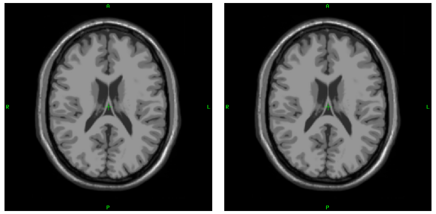


Multi-modal images: Measuring similarity

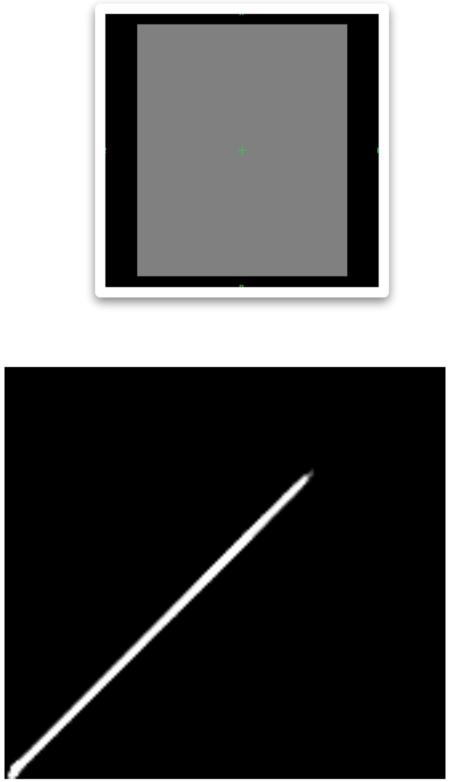


Joint Histogram, co-occurence counts …

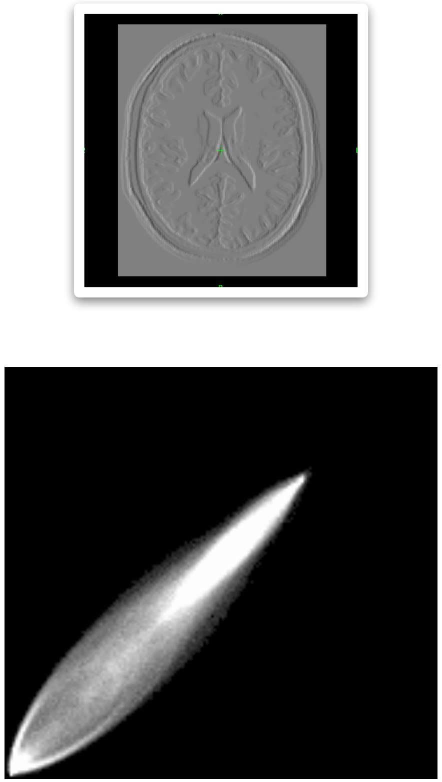
Measuring similarity: 2D histograms



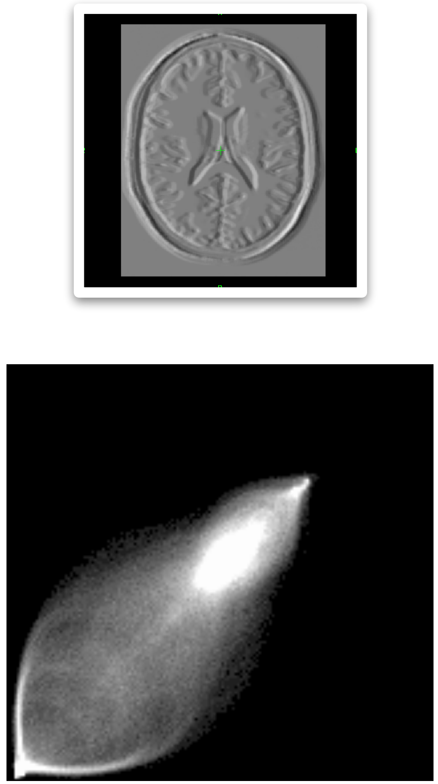
A mono-modality example (MR/MR)
(Same image twice!)



registered



mis-registered by 2mm



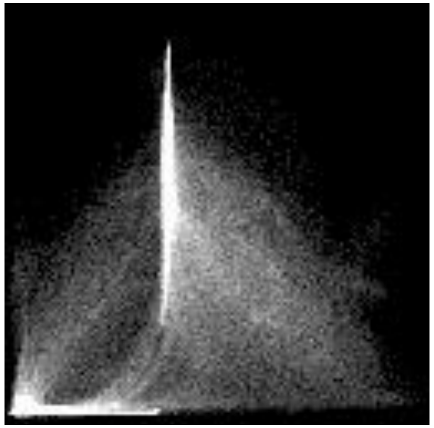
mis-registered by 5mm

Measuring similarity: 2D histograms

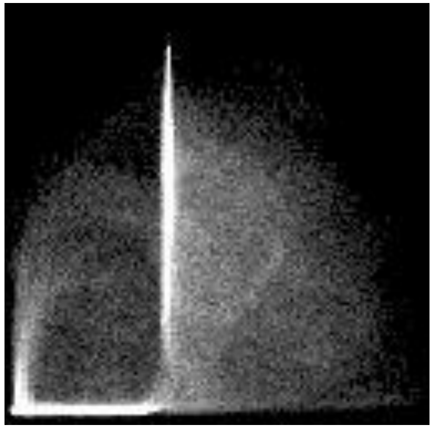
Multi-modality examples

MR/PET

registered



mis-registered by 2mm



mis-registered by 5mm

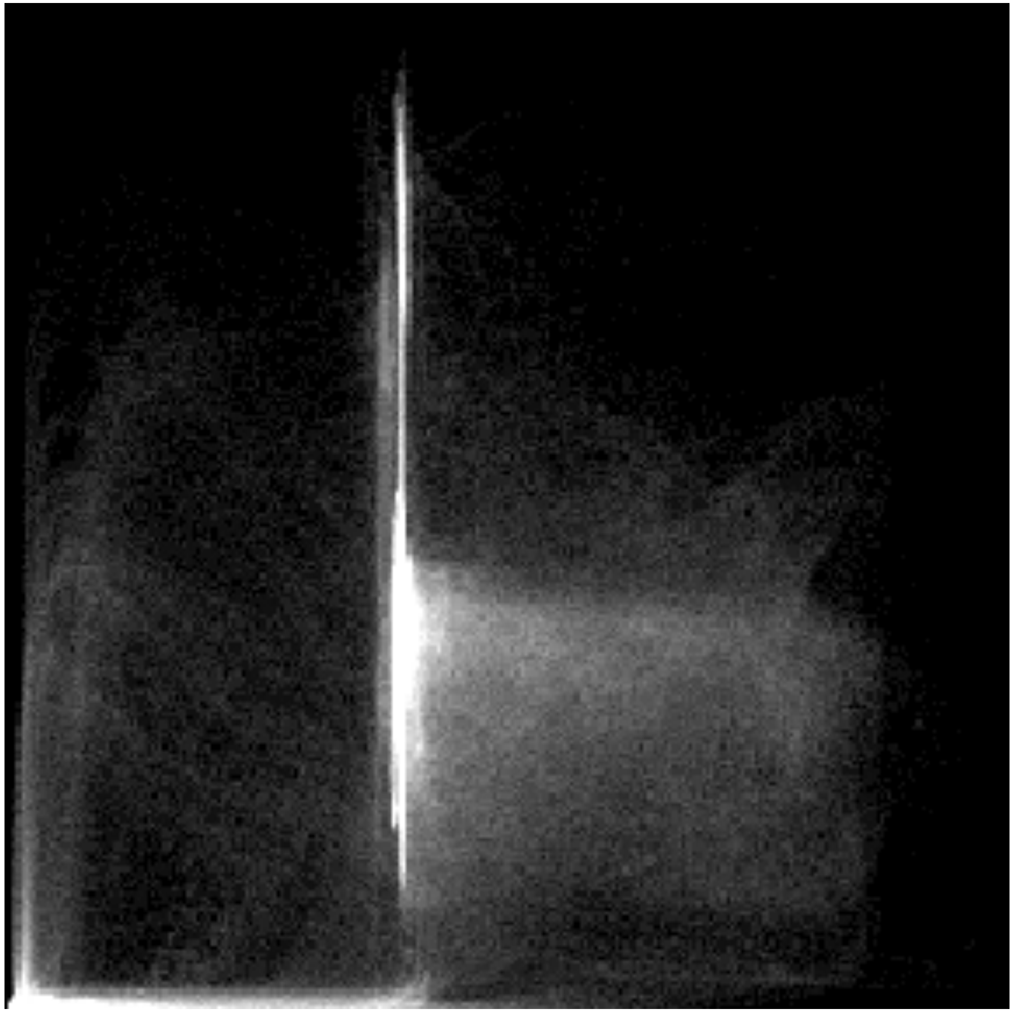
MR/CT

registered

mis-registered by 2mm

mis-registered by 5mm

Measuring similarity: 2D histograms



From images to probability distributions

- Frequency of corresponding intensity pairs can be interpreted in terms of probabilities

$$p(a,b) = \frac{h(a,b)}{N}$$

Joint probability of a voxel having value a in the first image and value b in the second image.

$$p(a) = \sum_b p(a,b)$$

Marginal probability of a voxel in the first image having value a

$$p(b) = \sum_a p(a,b)$$

Marginal probability of a voxel in the second image having grey value b

Voxel similarity using information theory

- Entropy (Shannon-Wiener)

$$H(A) = -\sum_{a} p(a) \log p(a)$$

describes the amount of information in image A.

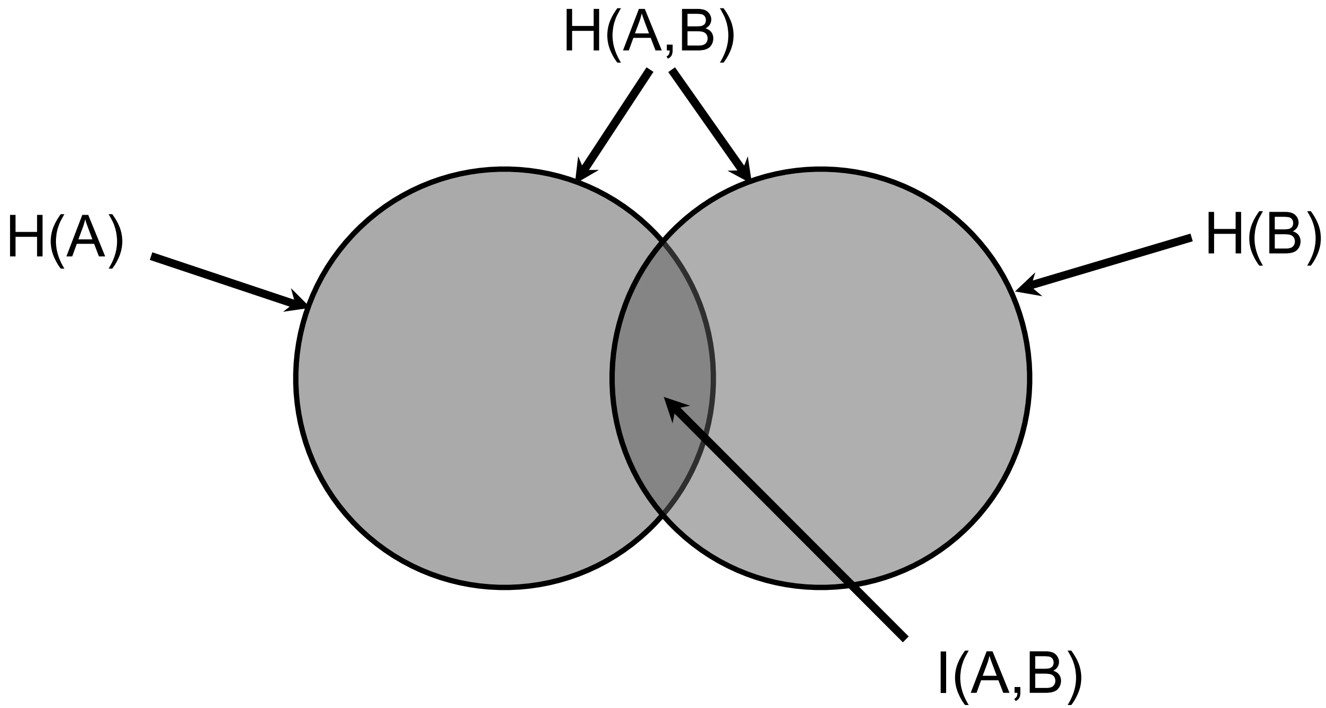
- Joint Entropy

$$H(A,B) = -\sum_{a} \sum_{b} p(a,b) \log p(a,b)$$

describes the amount of information in the combined images A and B.

Voxel similarity using information theory

- Venn diagram representation:

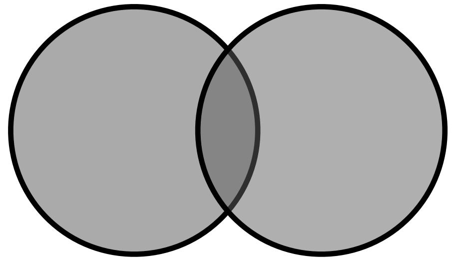


- Each circle represents the information content of one of the images - Intersection: shared information

Voxel similarity using information theory

- Mutual Information (Viola et al., 1995 and Collignon et al., 1995)

$$I(A,B) = H(A) + H(B) - H(A,B)$$



describes how well one image can be explained by another image.

- Mutual Information can be expressed in terms of marginal and joint probability distributions:

$$I(A,B) = -\sum_a \sum_b p(a,b) \log \frac{p(a,b)}{p(a)p(b)}$$

Summary: Voxel-based similarity measures

- Sums of squared differences

$$S = \frac{1}{N}\sum_i (I_A(\mathbf{x}_i) - I_B(\mathbf{T}(\mathbf{x}_i)))^2$$

- Cross-correlation

$$S = \frac{\sum \left(I_A(\mathbf{x}) - \mu_A\right)\left(I_B(\mathbf{T}(\mathbf{x})) - \mu_B\right)}{\sqrt{\left(\sum \left(I_A(\mathbf{x}) - \mu_A\right)^2\right)\left(\sum \left(I_B(\mathbf{T}(\mathbf{x})) - \mu_B\right)^2\right)}}$$

Mono-modal

- Mutual information (or variants)

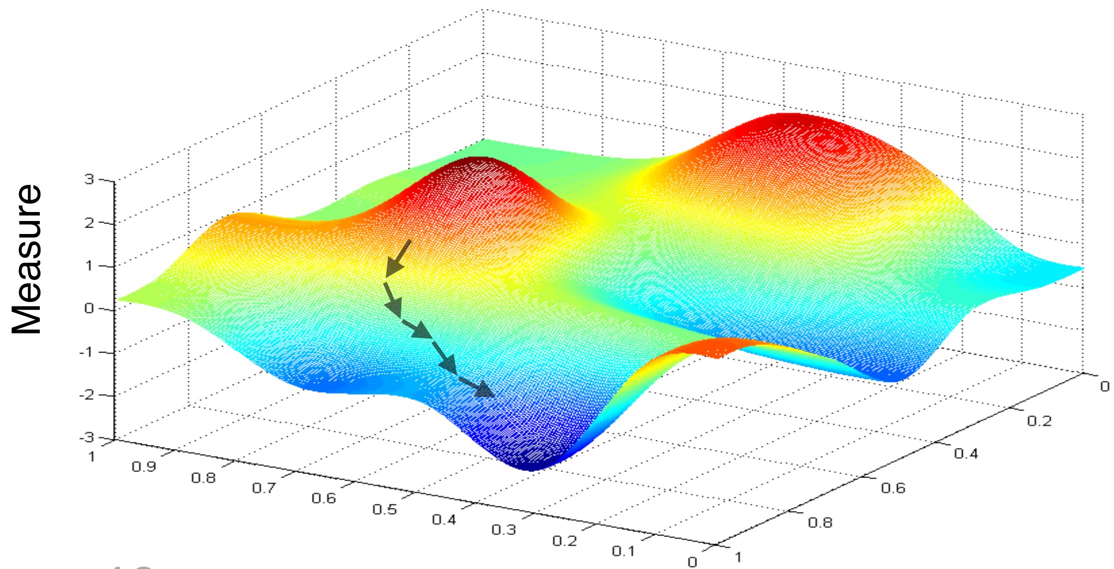
$$S = -\sum_a \sum_b p(a,b) \log \frac{p(a,b)}{p(a)p(b)}$$

Multi-modal

Optimisation of voxel-similarity measures

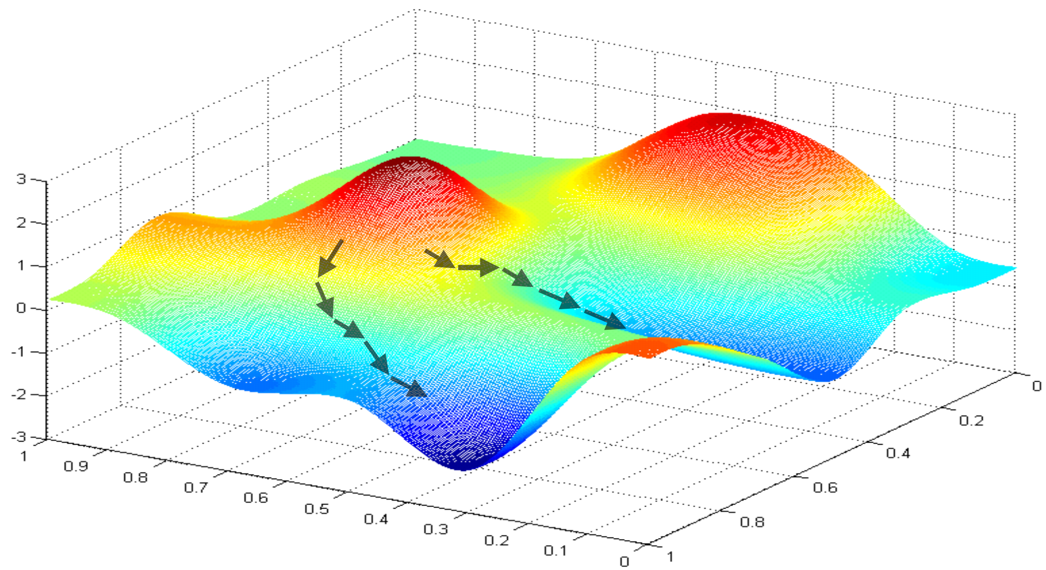
- Optimisation of voxel-similarity measures normally requires iterative (gradient-based) techniques, E.g.
  - Steepest gradient descent
  - Conjugate gradient descent
  - More recently, discrete gradient free optimisation techniques have been used (e.g. Glocker et al, IPMI 2007)

Measure ‘Surface’

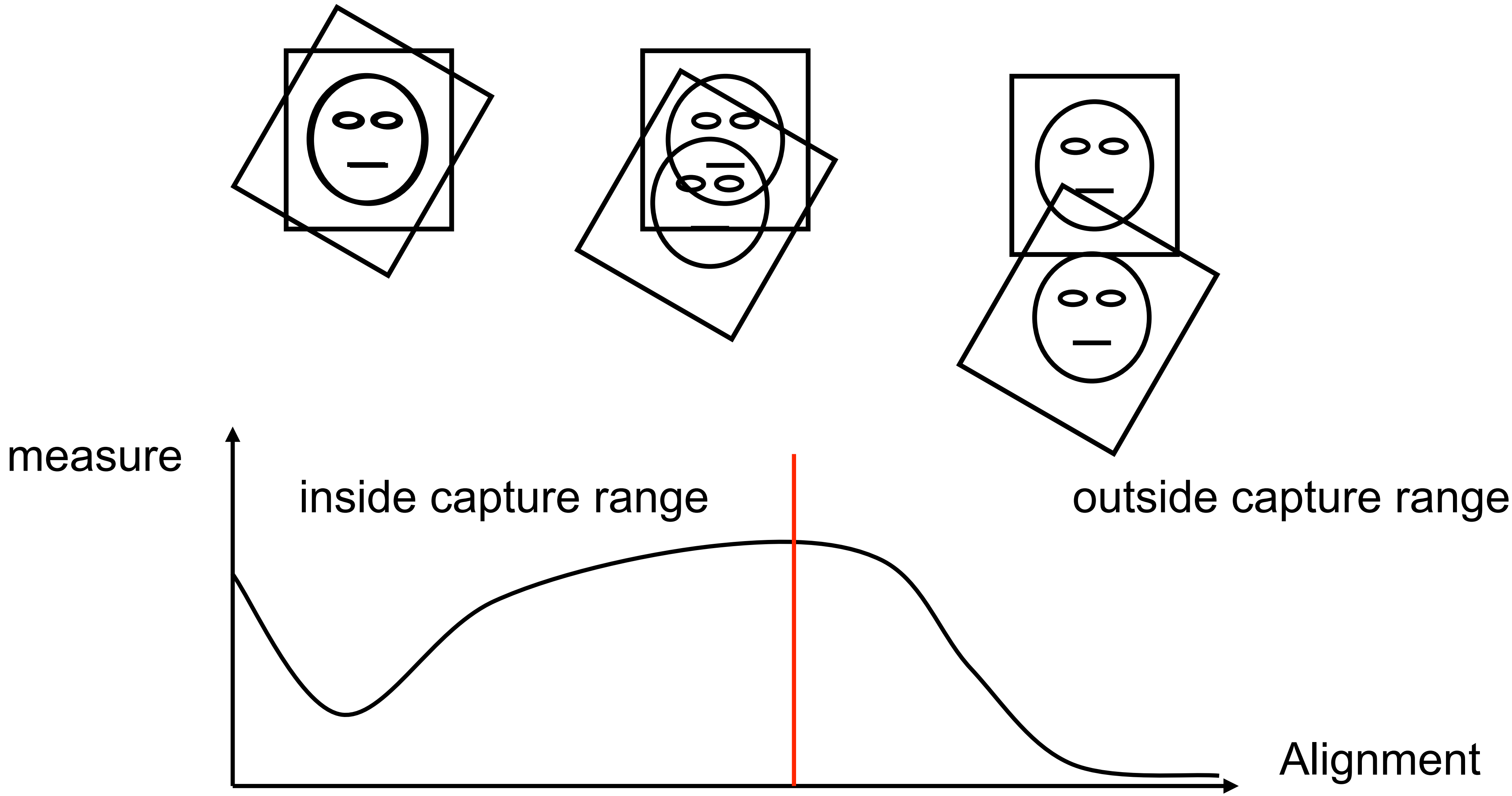


Optimisation of voxel-similarity measures

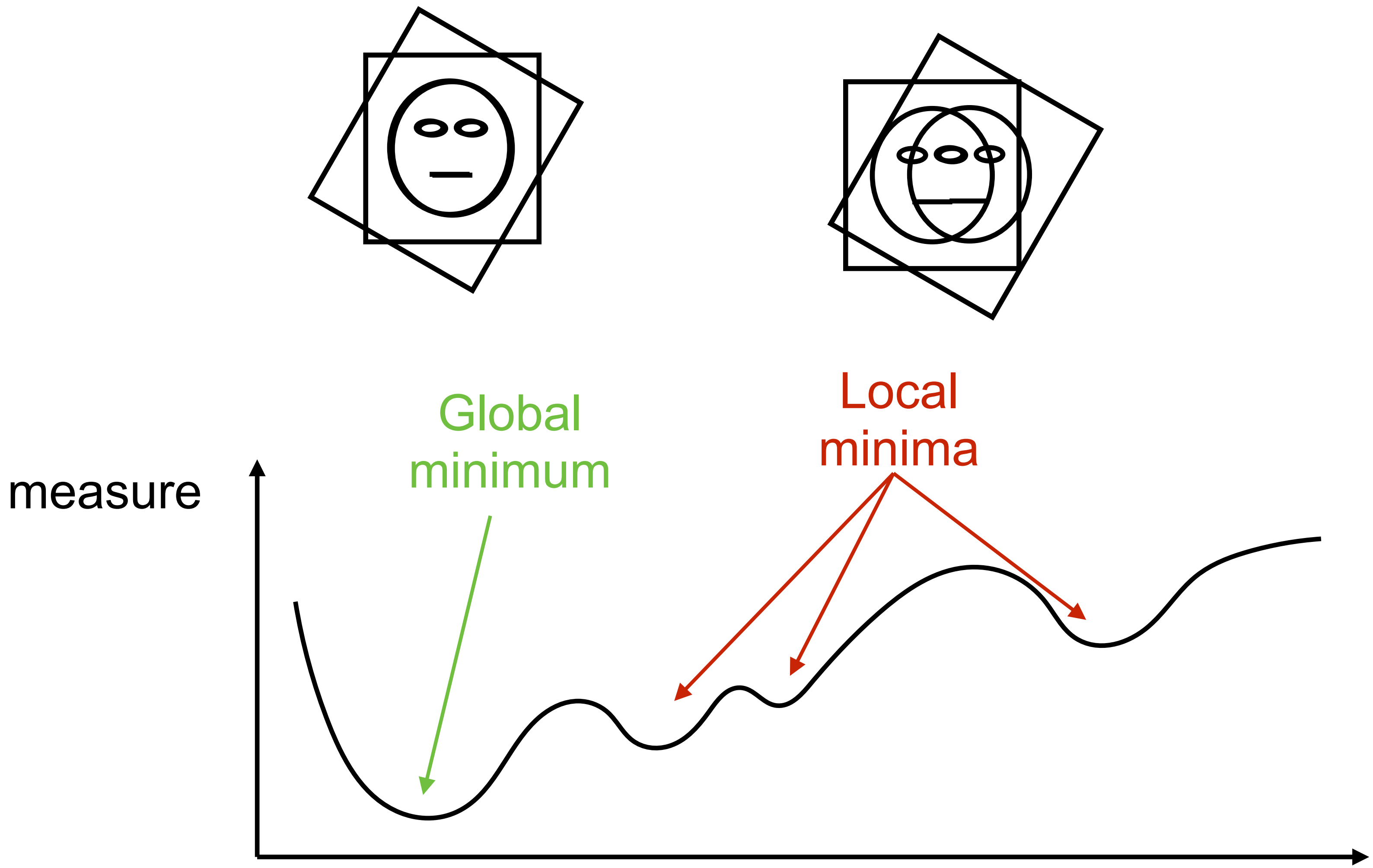
- In general:
  - Global optimisation schemes are not feasible for image registration
  - Local optimisation schemes are much more efficient but will get trapped in local optima
  - Registration has a limited ‘capture range’



Optimisation of voxel-similarity measures

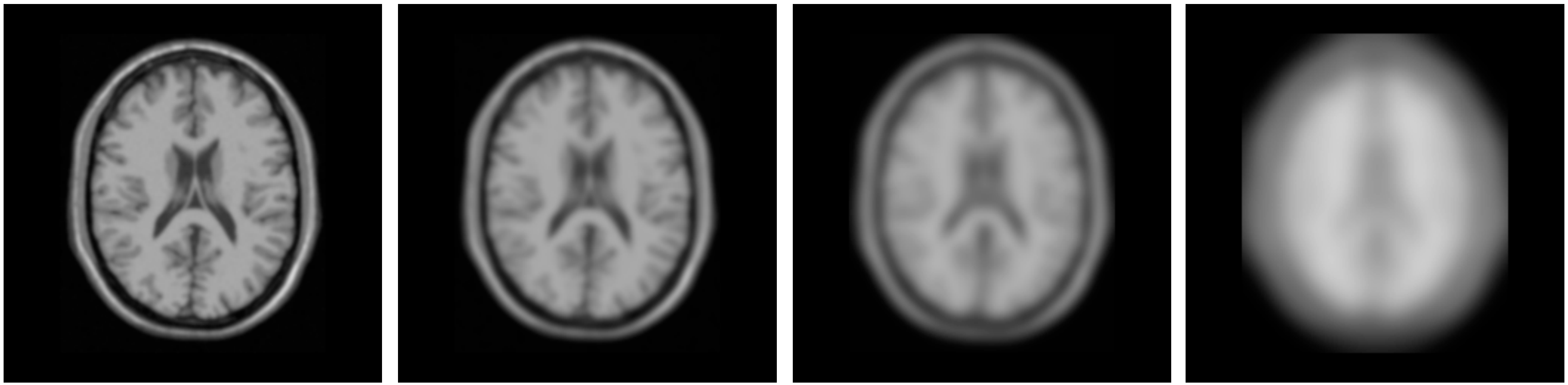


Optimisation of voxel-similarity measures



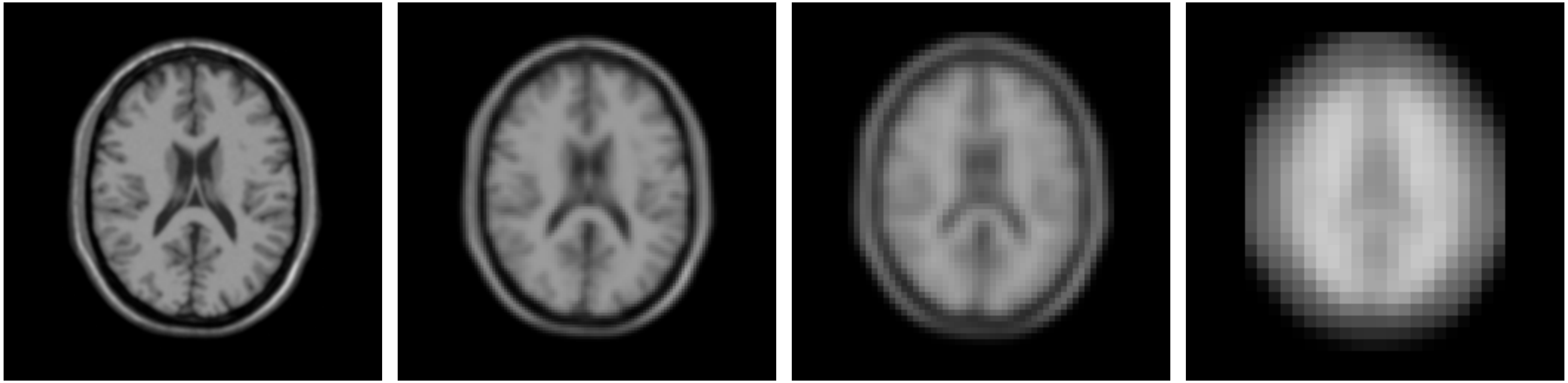
Multi-scale optimisation

Capture range can be improved with multi-scale methods:

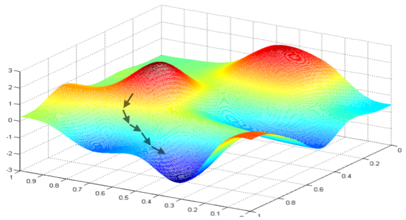


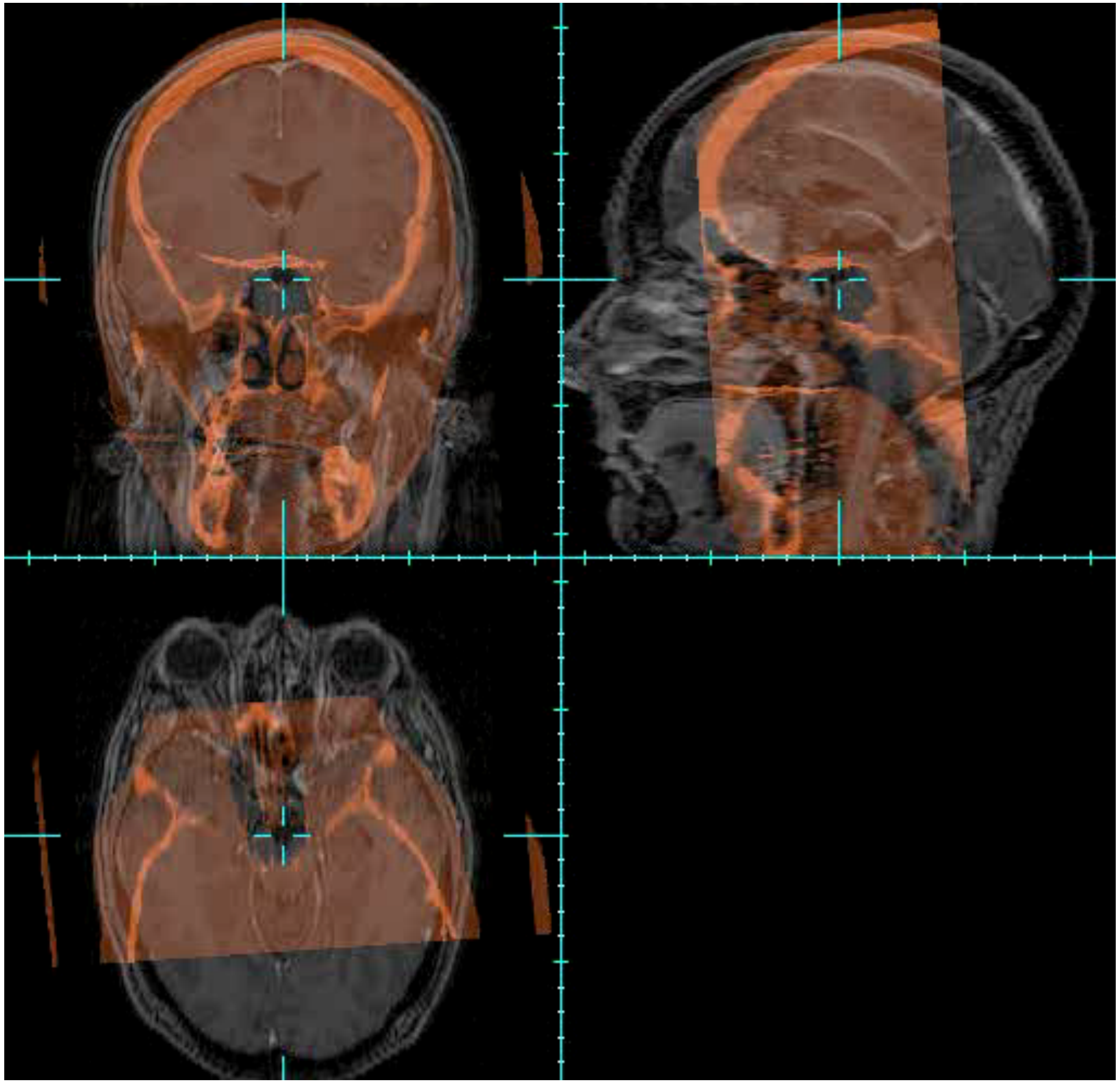
Multi-resolution optimisation

Registration can be accelerated with multi-resolution techniques:



Rigid registration using Mutual Information

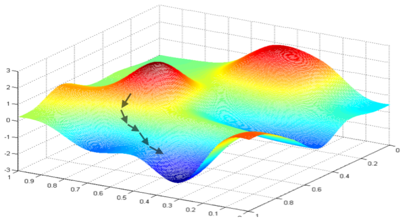




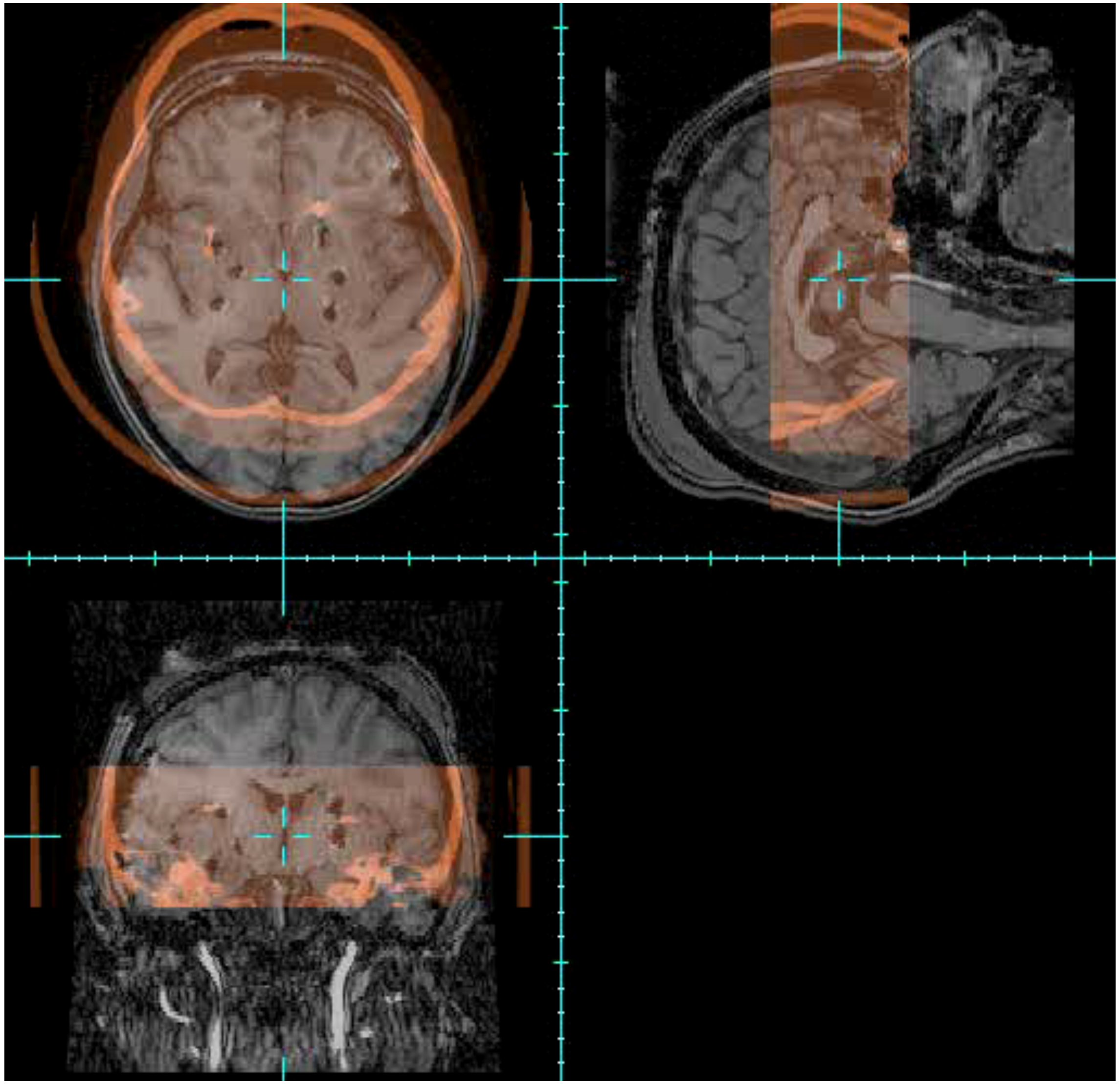
Visualise progression of parameter estimates

Thanks to
Colin Studholme

Rigid registration using Mutual Information



Visualise progression of parameter estimates



Thanks to
Colin Studholme

Non-rigid registration

- Rigid registration is appropriate for
  - brain (constrained by skull)
  - bone (neck, vertebrae)
- Affine registration is appropriate
  - if not all image acquisition parameters are known:
    - unknown voxel sizes
    - unknown gantry tilt
  - if scale changes are expected:
    - growth
    - inter-subject registration

Limited applicability except as initialisation for non-rigid registration

Non-rigid registration in neuroimaging

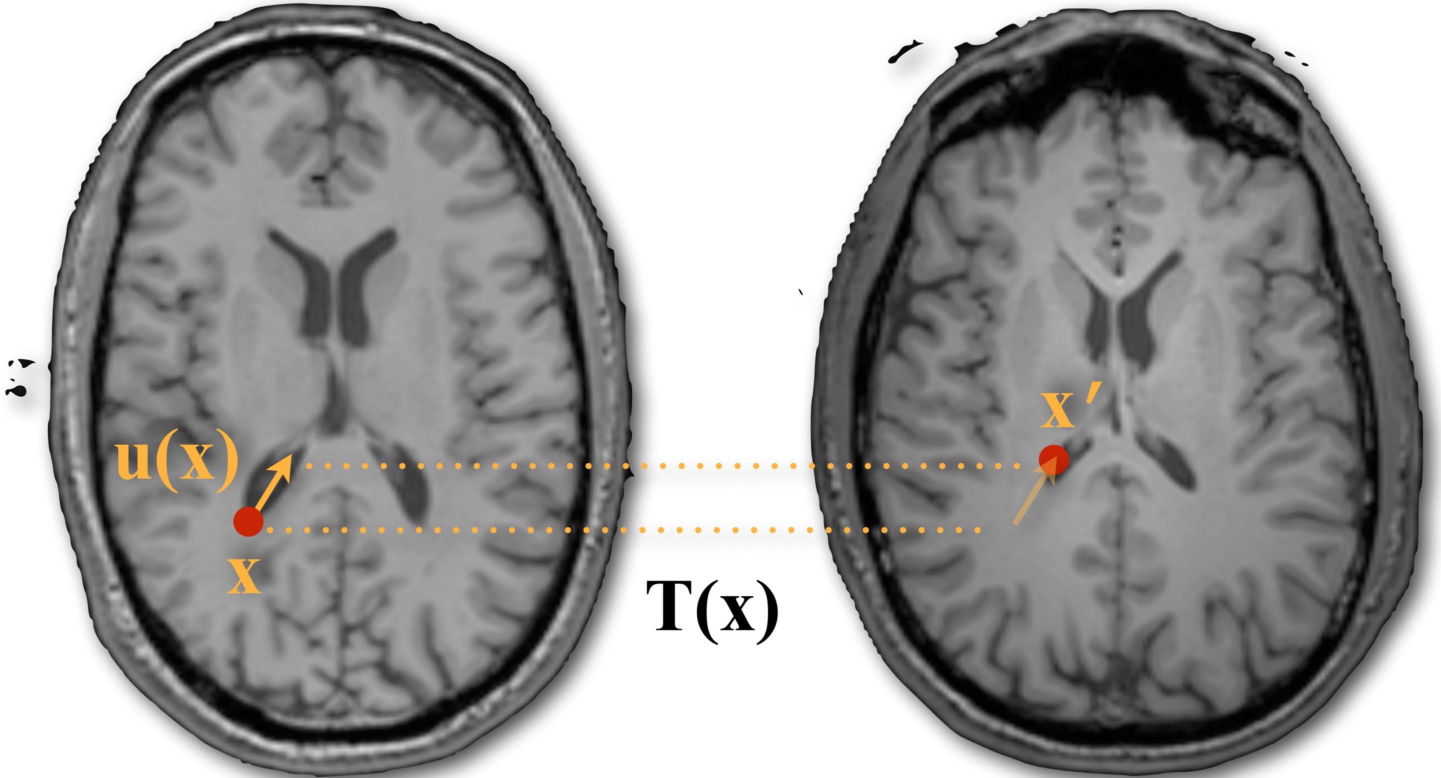
A number of application areas:

- Motion:
  - Compensation for tissue deformation
    - Brain deformation during neurosurgery
    - Fusion of functional and anatomical information (MR/PET, etc.)
- Longitudinal or cross-sectional studies:
  - Quantification of change over time,
    - brain atrophy
    - brain growth
  - Quantification of differences between populations
    - voxel-based morphometry (low-dimensional transformations)
    - deformation-based morphometry (high-dimensional transformations)
- Atlas-based studies
  - Segmentation via atlas propagation

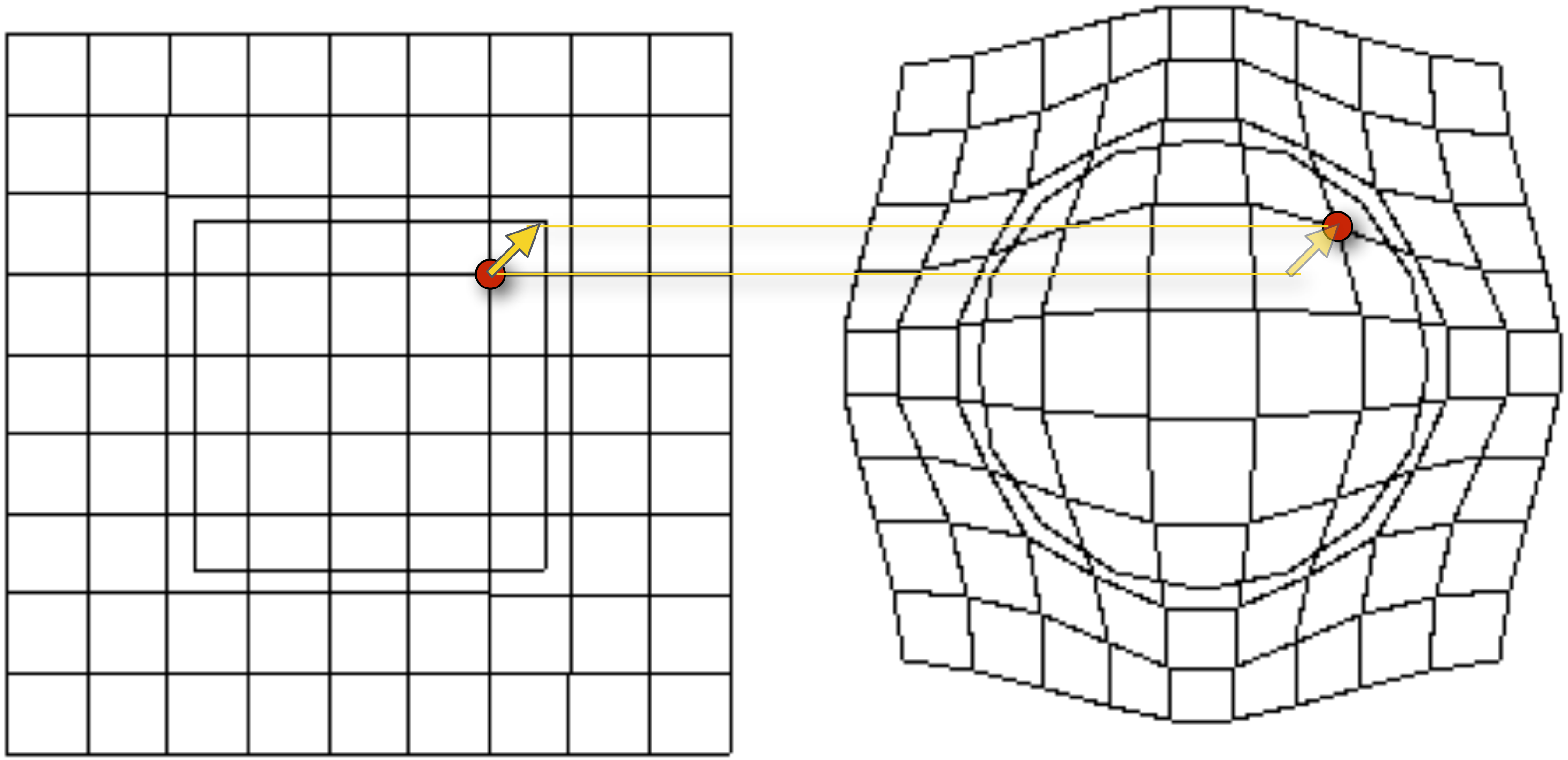
Representing deformations

- Transformation $\mathbf{T}(\mathbf{x}) = \mathbf{x}'$ defines spatial relationship between two images:

$$\mathbf{x}' = \mathbf{T}(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$$



Representing deformations



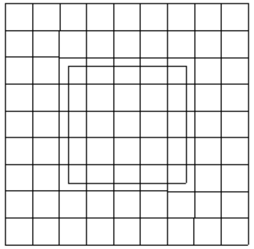
Before deformation

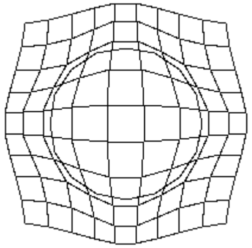
$$\mathbf{x}$$

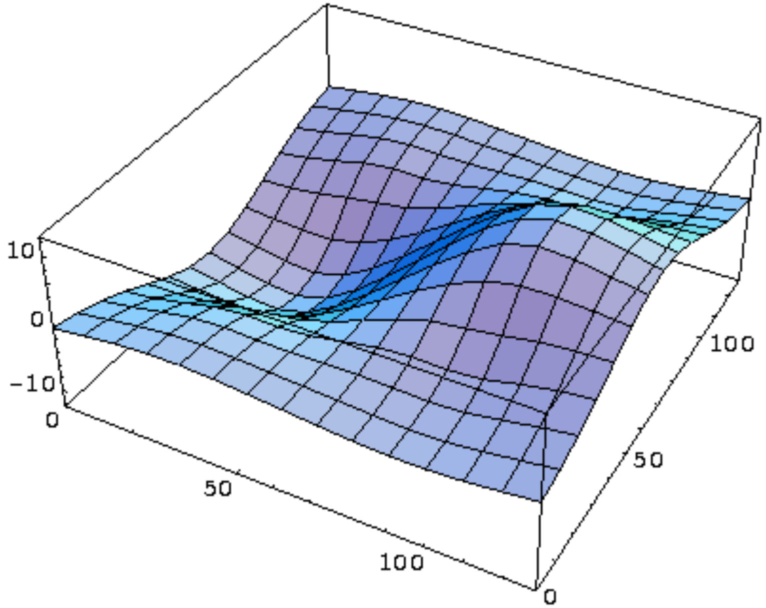
After deformation

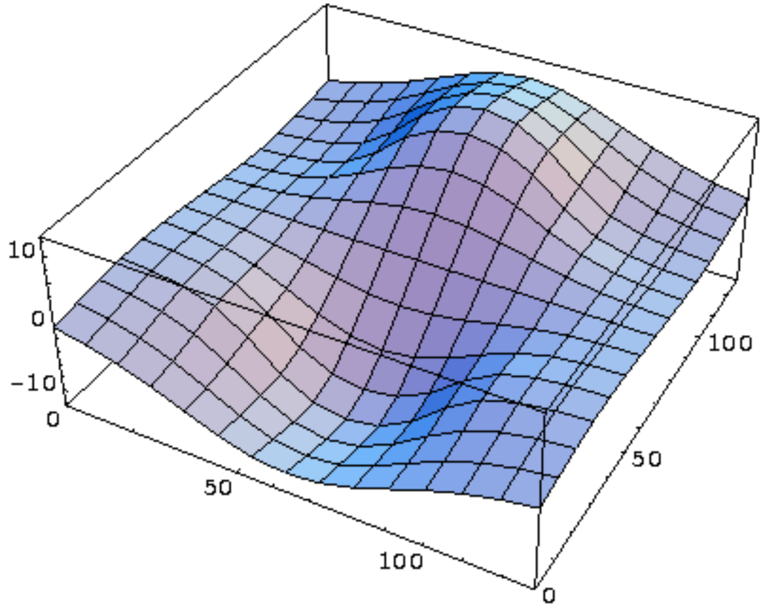
$$\mathbf{x} + \mathbf{u}(\mathbf{x})$$

Representing deformations









Displacement in the horizontal direction

$$u_x(x, y)$$

Displacement in the vertical direction

$$u_y(x, y)$$

Types of non-rigid transformations

- Parametric transformations
  - affine transformations
  - polynomial transformations
    - linear
    - quadratic
    - cubic
  - spline-based transformations

#DOFs << #voxels

- Non-parametric transformations
  - optical flow type registration (e.g. Demons)
  - elastic registration
  - fluid or geodesic registration

#DOFs = #voxels

Types of non-rigid transformations

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#DOFs << #voxels

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#DOFs = #voxels

Non-rigid transformations

- Recall affine transformation: linear function of (x,y,z).
- It has 12 DOF

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{12} & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$(x', y', z')^T = \mathbf{T}(x, y, z)$$

- Can get non-rigid version with higher order polynomials:
  - 2nd order $(x', y', z')^T = \mathbf{T}(x, y, z, x^2, y^2, z^2, xy, xz, yz)$
  - For a linear function $\mathbf{T}$
  - Needs 30 parameters (30 DOF)
  - Higher orders: (3rd, 60 DOF), (4th,105 DOF), (5th,168 DOF)

Non-rigid transformations

- But higher order polynomials have problems:
  - Can model only **global** shape changes, not **local** shape changes
  - Modifying a single parameter has an effect globally
  - Higher order polynomials introduce artifacts such as oscillations

- Has been used for registration
  - Woods et al., Journal of Computer Assisted Tomography, 22(1), 1998, pp 153-165

Non-rigid transformations

- Alternative to polynomials: Use linear combinations of basis functions θ_i

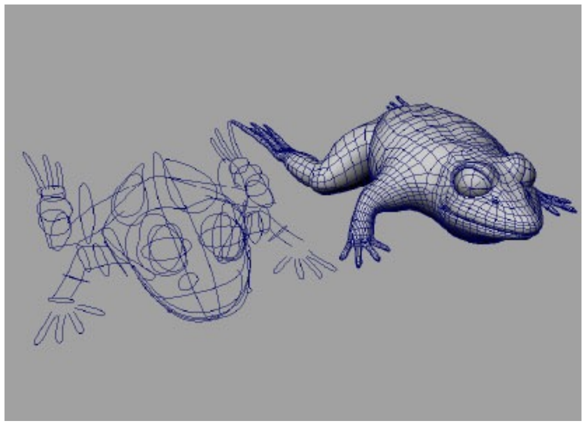
$$\mathbf{T}(x,y,z) = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} r_{10} & \cdots & r_{1n} \\ r_{20} & \cdots & r_{2n} \\ r_{30} & \cdots & r_{3n} \\ 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \theta_0(x,y,z) \\ \vdots \\ \theta_{n-1}(x,y,z) \\ 1 \end{pmatrix}$$

Non-rigid transformations

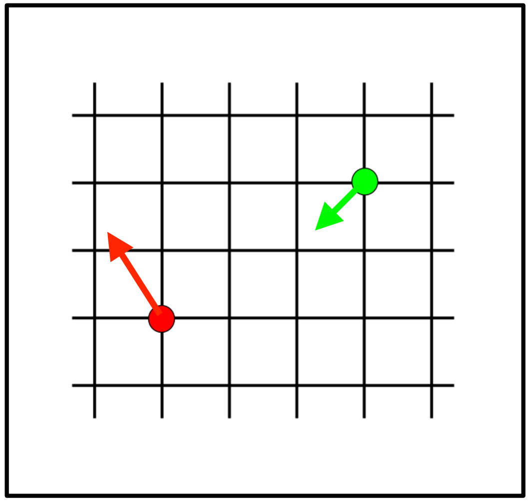
- Basis functions may be
  - Trigonometric basis functions (used by SPM)
  - Wavelet basis functions
- Used for registration
  - Amit, et al., Structural Image Restoration Through Deformable Templates, Journal of the American Statistical Association, 86, 1991
  - Ashburner & Friston, Nonlinear spatial normalization using basis functions, Human Brain Mapping 7, 1999

Splines for modelling non-rigid transformations

- Engineering:
  - Splines are long flexible strips of wood or metal which were deformed by attaching clamps or weights along their length
- Mathematics:
  - Splines are a tool used to ***approximate*** or ***interpolate*** functions from scattered data:
  - 1D: curves
  - 2D: surfaces
  - 3D: volumes



Non-rigid registration
Interpolating splines

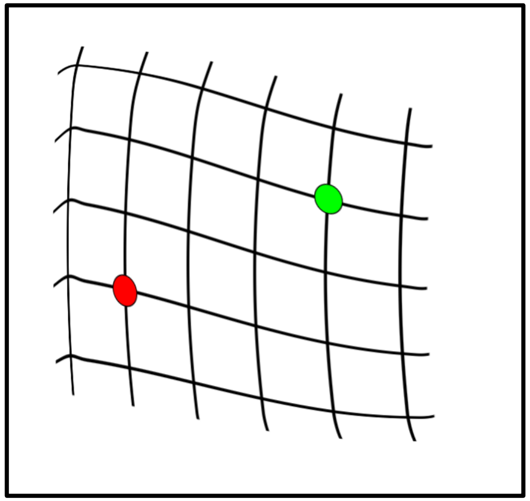


- Basic idea:
  - identify a set of points ϕ_i in image A
  - identify the corresponding points ϕ'_i in image B
  - find a spline transformation which
    - Interpolates the displacement field at the points $\phi_i \rightarrow \phi'_i$

$$\mathbf{T}(\phi_i) = \phi'_i \qquad i = 1, \ldots, n$$

    - Produce smooth displacement field everywhere else

  - points can be:
    - anatomical or geometrical landmarks
    - pseudo landmarks or control points



Non-rigid registration
Thin-plate splines

- Thin-plate splines can be defined as a linear combination of radial basis functions θ :

$$t(x,y,z) = a_1 + a_2 x + a_3 y + a_4 z + \sum_{j=1}^{n} b_j \, \theta\left(|\phi_j - (x,y,z)^T| \right)$$

$$\theta(r) = \begin{cases} r^2 \log(r^2) & \text{in 2-D} \\ r & \text{in 3-D} \end{cases}$$

Need to find the a's and the b's
ϕ_i and $\mathbf{T}(\phi_i) = \phi'_i$ are given

- A transformation between two images can be defined by three separate thin-plate splines:

$$\mathbf{T}(x,y,z) = \left(t_1(x,y,z) \, , \; t_2(x,y,z) \, , \; t_3(x,y,z) \right)^T$$

Non-rigid registration
Thin-plate splines

- Solve for a's and b's by writing interpolation conditions $\mathbf{T}(\phi_i) = \phi'_i$ in matrix form

$$\begin{pmatrix} \Theta & \Phi \\ \Phi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{b} \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} \Phi' \\ \mathbf{0} \end{pmatrix}$$

$\mathbf{\Phi}, \mathbf{\Phi}'$: n x 4 matrices with point data

$\mathbf{a}$: 4 x 3 matrix containing the affine coefficients

$\mathbf{b}$: n x 3 matrix containing the non-affine coefficients

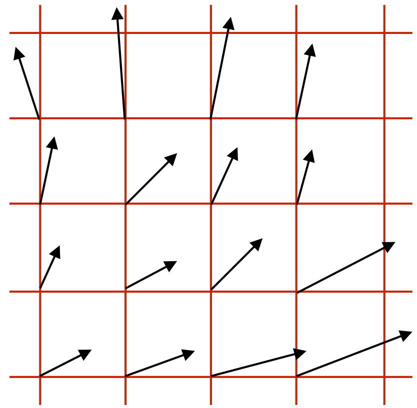
$\mathbf{\Theta}$: a n x n matrix with

$$\Theta_{ij} = \theta\left(\left|\phi_i - \phi_j\right|\right)$$

Non-rigid registration
Thin-plate splines

- Used for registration:
  - With points: (Bookstein, Principal Warps: Thin-Plate Splines and the Decomposition of Deformations, PAMI, 11, 1989, also Goshtasby, IEEE Geoscien. & Rem. Sensing, 1998)
  - With voxel similarity measures - (Meyer, Medical image analysis 1, 1997)
- Advantages:
  - Control points can have arbitrary spatial distribution
- Disadvantages:
  - Control points have global influence since the radial basis function has infinite support

Non-rigid registration
B-splines



Free-Form Deformations (FFDs):

- A common technique in Computer Graphics for modelling 3D deformable objects
- Parametrised by a regular $n_x \times n_y \times n_z$ mesh of control points $\mathbf{\Phi}$, each with a vector and with a spacing of δ
- Deform underlying object by manipulating the control points
- Control points ***only affect local neighbourhood*** (compact support)

Non-rigid registration
B-splines

- FFDs based on B-splines can be expressed as a 3D tensor product of 1D B-splines:

$$\mathbf{T}(x,y,z) = \sum_{l=0}^{3}\sum_{m=0}^{3}\sum_{n=0}^{3} B_l(r)B_m(s)B_n(t)\mathbf{\Phi}_{i+l,j+m,k+n}.$$

where

$$i = \lfloor x/\delta \rfloor - 1 \quad j = \lfloor y/\delta \rfloor - 1 \quad k = \lfloor z/\delta \rfloor - 1$$

$$r = x/\delta - \lfloor x/\delta \rfloor \quad s = y/\delta - \lfloor y/\delta \rfloor \quad t = z/\delta - \lfloor z/\delta \rfloor$$

- Summation only over control points in local neighbourhood of (x,y,z) - **compact support**.

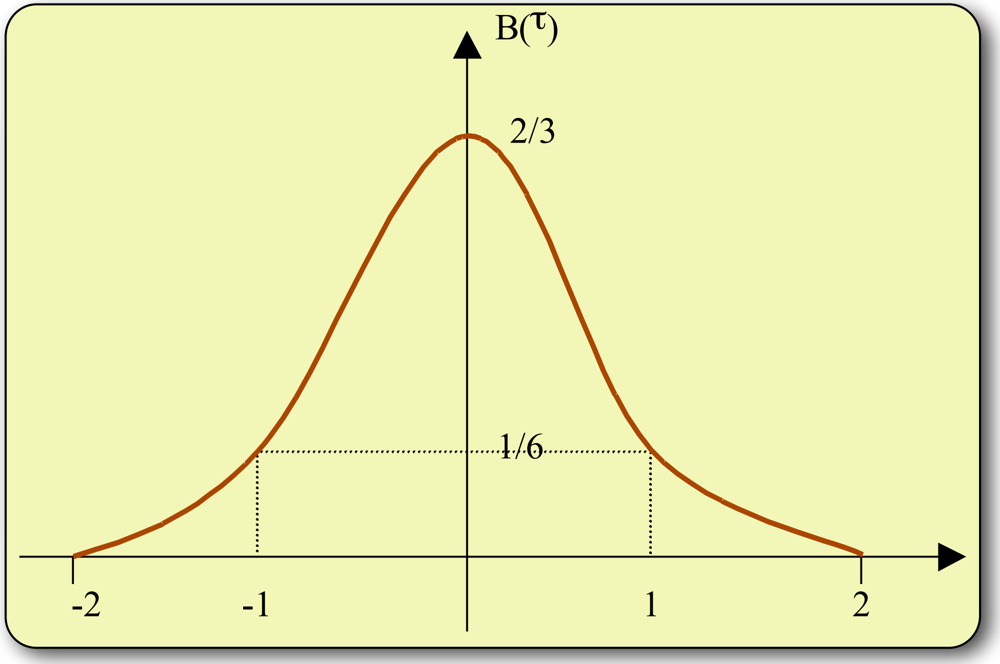
Non-rigid registration
B-splines

- B_i corresponds to the pieces of the B-spline basis function:

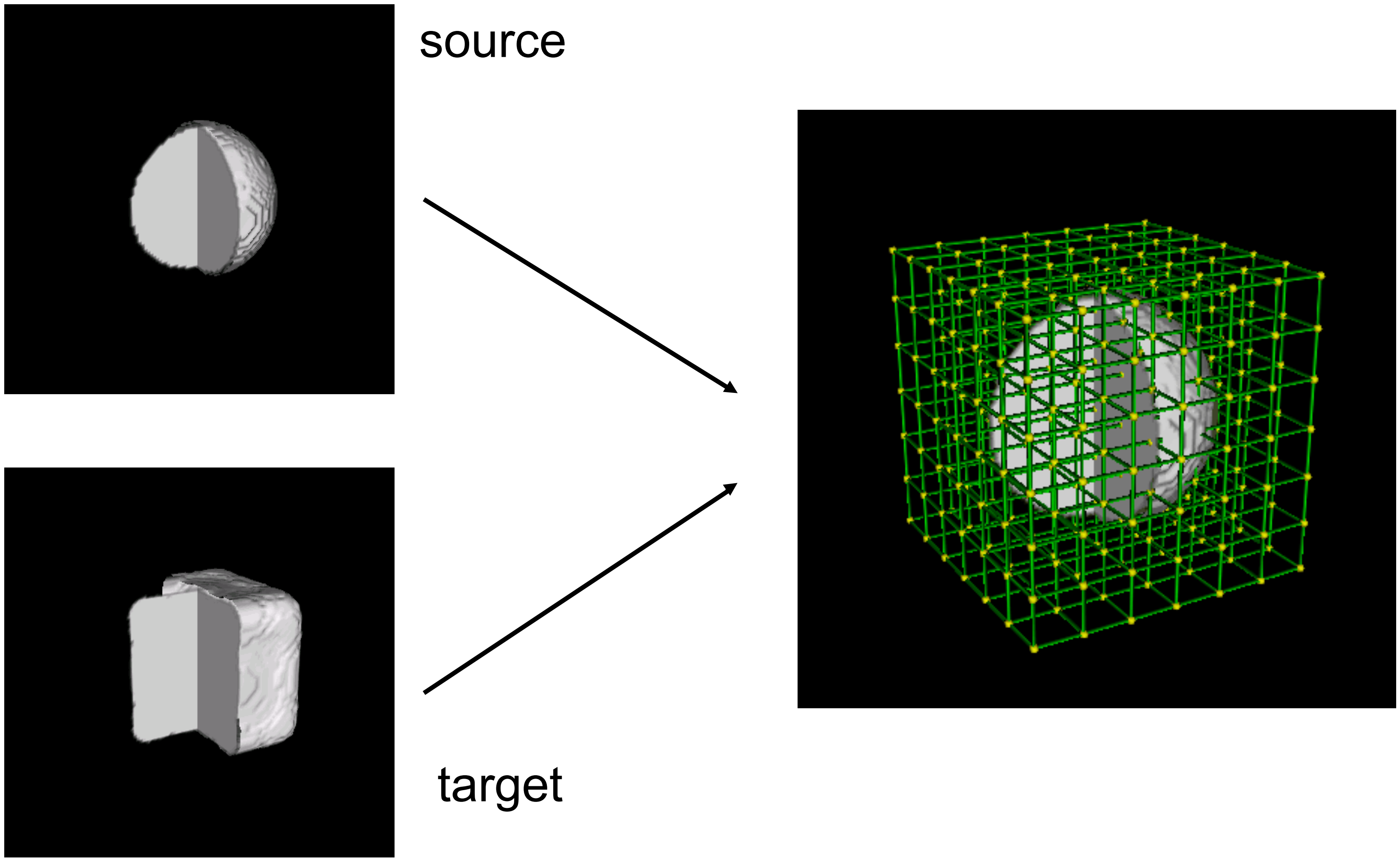
$$B_0(u) = \frac{(1-u)^3}{6} \qquad B_2(u) = \frac{-3u^3+3u^2+3u+1}{6}$$

$$B_1(u) = \frac{3u^3-6u^2+4}{6} \qquad B_3(u) = \frac{u^3}{6}$$

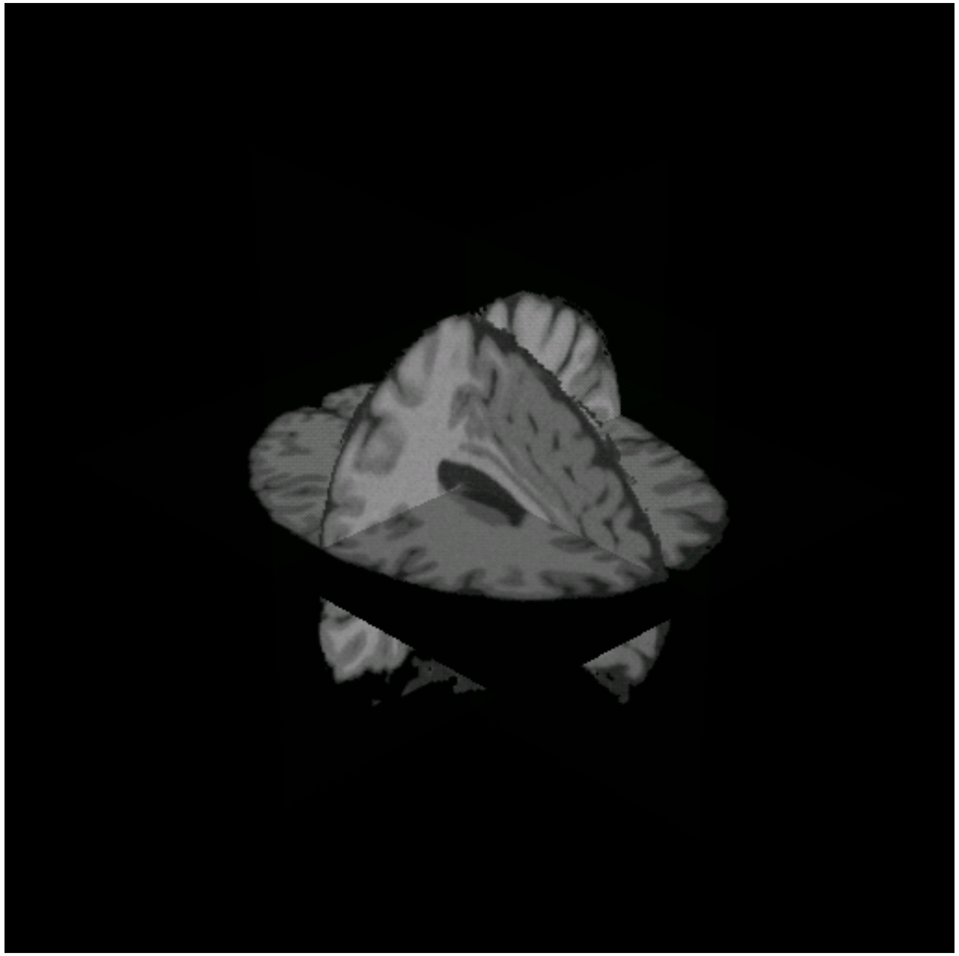
$$B(t) = \begin{cases} 0 & |t| > 2 \\ B_3(t+2) & -2 < t \leq -1 \\ B_2(t+1) & -1 < t \leq 0 \\ B_1(t) & 0 < t \leq 1 \\ B_0(t-1) & 1 < t < 2 \end{cases}$$

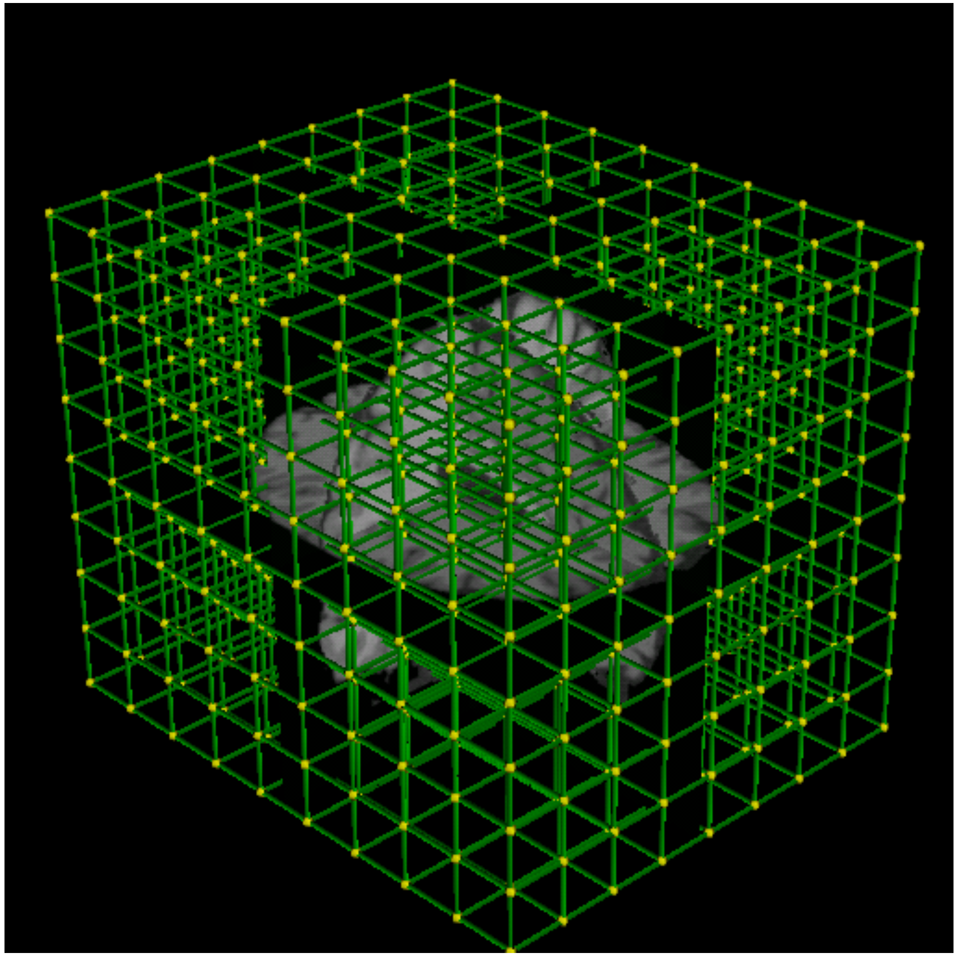


Non-rigid registration
Free-form deformations



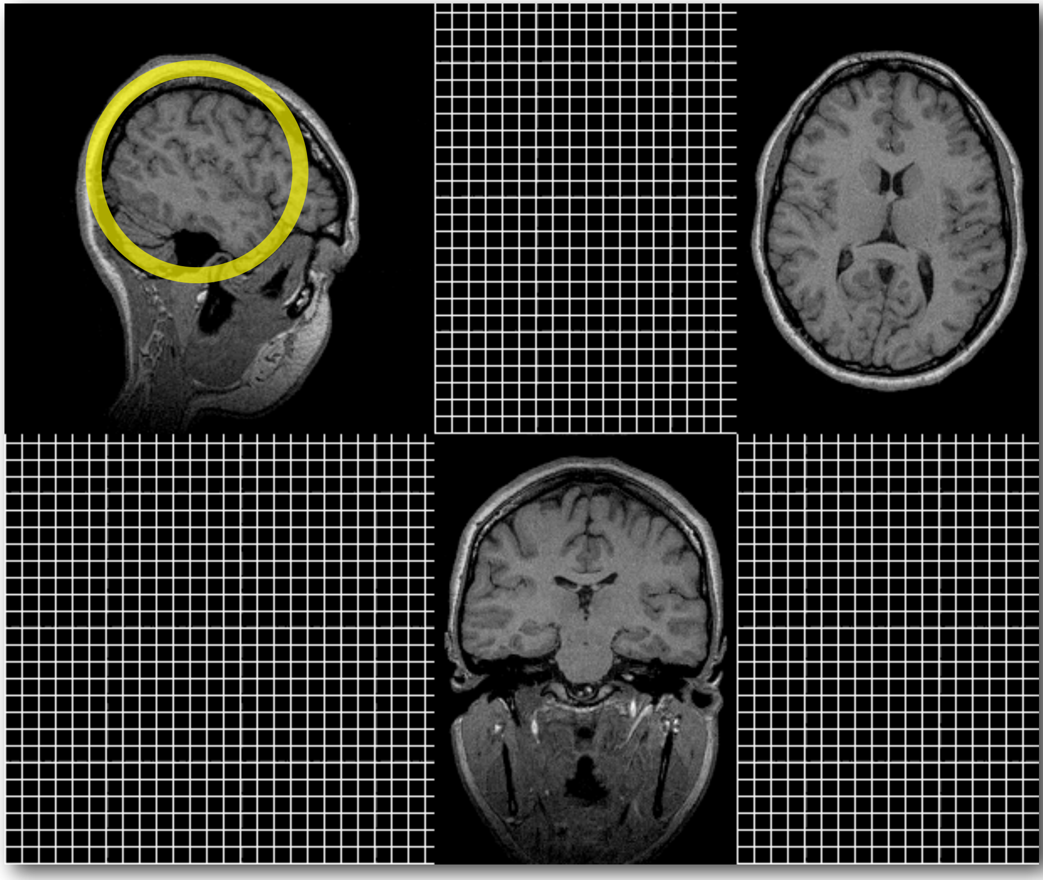
Non-rigid registration
Free-form deformations



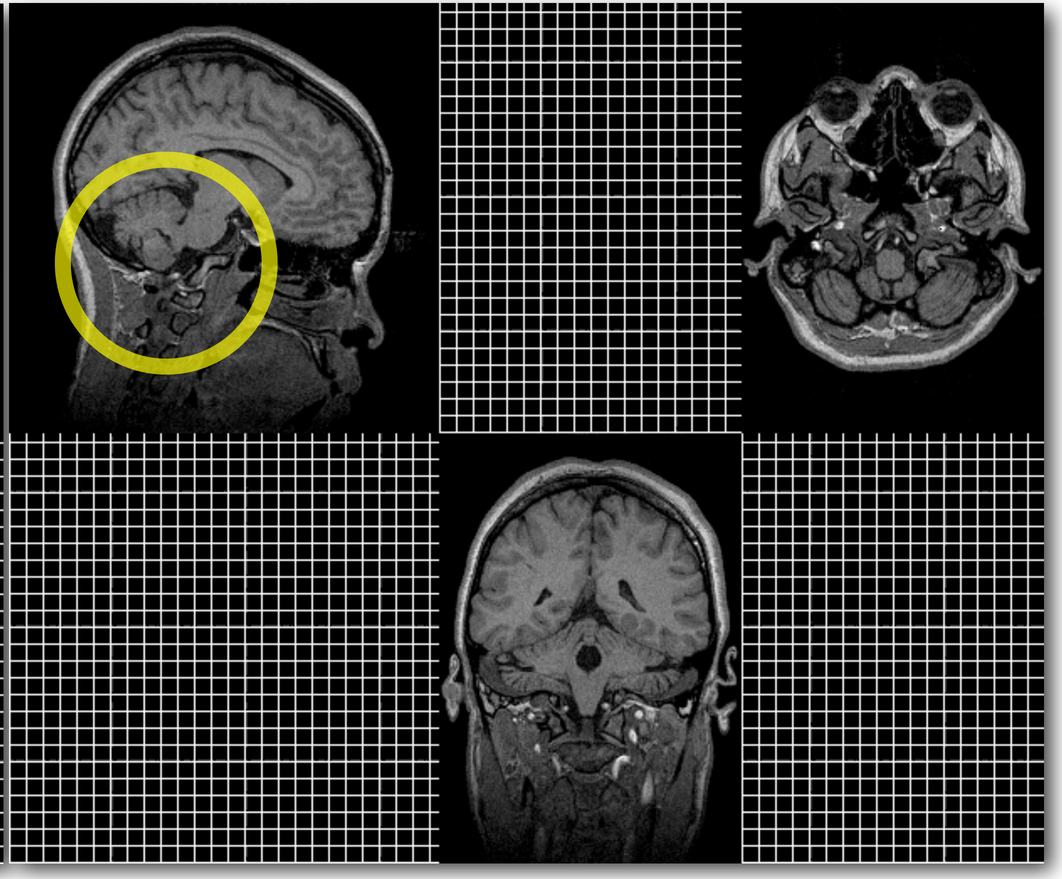


Non-rigid registration
Free-form deformations

Displacement of 1 control point along 1 axis



5 mm spacing



20 mm spacing

Thanks to Marc Modat

Image registration: Adding regularisation

- Non-rigid registration problem is ill-posed: there are many possible solutions.
- So regularisation is often included in the cost function to obtain realistic deformations:

$$C = S + \alpha P$$

Similarity Penalty

- Bayesian interpretation:
  - Similarity term → likelihood
  - Penalty term → prior probability which expresses our knowledge about what kind of deformations we expect

Image registration: Adding regularisation

- Diffusion-like regularisation

$$P = \iiint \left(\frac{\partial \mathbf{u}}{\partial x}\right)^2 + \left(\frac{\partial \mathbf{u}}{\partial y}\right)^2 + \left(\frac{\partial \mathbf{u}}{\partial z}\right)^2$$

- Curvature-like regularisation

$$P = \iiint \left(\frac{\partial^2 \mathbf{u}}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \mathbf{u}}{\partial y^2}\right)^2 + \left(\frac{\partial^2 \mathbf{u}}{\partial z^2}\right)^2 + 2\left[\left(\frac{\partial^2 \mathbf{u}}{\partial xy}\right)^2 + \left(\frac{\partial^2 \mathbf{u}}{\partial xz}\right)^2 + \left(\frac{\partial^2 \mathbf{u}}{\partial yz}\right)^2\right]$$

- Others like volume preservation (see next slide)

Key property of non-rigid transformations

- Jacobian determinant

$$\mathrm{Jac}(\mathbf{u}) = \det \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

$\mathrm{Jac}(\mathbf{u})$	Properties
$\mathrm{Jac}(\mathbf{u}) = 1$	volume preservation
$\mathrm{Jac}(\mathbf{u}) < 1$	local contraction
$\mathrm{Jac}(\mathbf{u}) > 1$	local expansion
$\mathrm{Jac}(\mathbf{u}) = \infty$	tearing
$\mathrm{Jac}(\mathbf{u}) < 0$	folding

Regularization penalties revisited …

- Local volume preservation (Rohlfing, TMI 2003):

$$P = \int \left|\log\left(\, J(\mathbf{x}) \,\right)\right| \, d\mathbf{x}$$

- Local rigidity (Loeckx, MICCAI 2004):

$$P = \int \left\|\log\left(\, J^T(\mathbf{x}) J(\mathbf{x}) - \mathbf{I} \,\right)\right\| \, d\mathbf{x}$$

- Others are possible, e.g. for topology preservation

Types of non-rigid transformations

- Parametric transformations
  - affine transformations
  - polynomial transformations
    - linear
    - quadratic
    - cubic
  - spline-based transformations

#DOFs << #voxels

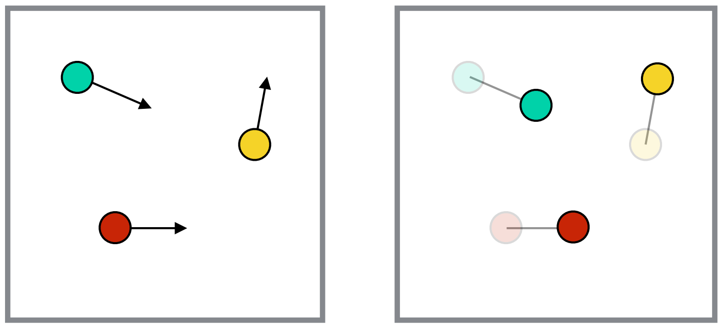
- Non-parametric transformations
  - optical flow type registration (e.g. Demons)
  - elastic registration
  - fluid or geodesic registration

#DOFs = #voxels

Registration using optical flow

- Optical flow is a well-known computer vision technique.
- Often used to register successive frames in video sequences
- Basic assumption is brightness constancy
  - Point at (x,y,t) 'flows' to $(x+\Delta x,\ y+\Delta y,\ t+\Delta t)$ and keeps its brightness

$$I(x, y, t) \;=\; I(x + \Delta x, y + \Delta y, t + \Delta t)$$



Registration using optical flow

- Use Taylor series and ignore higher order terms, OF equation becomes:

$$\frac{\partial I}{\partial x}\frac{dx}{dt}+\frac{\partial I}{\partial y}\frac{dy}{dt}+\frac{\partial I}{\partial z}\frac{dz}{dt}+\frac{\partial I}{\partial t}=0$$

- Or

$$\nabla I \cdot \mathbf{u} = I - J$$

I-J : Temporal difference between images

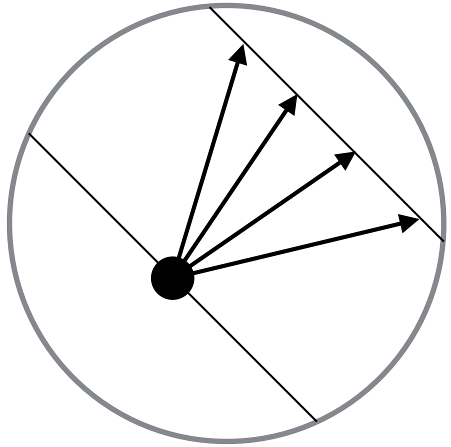
∇I : Spatial gradient of the image

$\mathbf{u}$: Displacement (‘flow’) between images

To estimate the displacement field, we need additional constraints (i.e. smoothness)

Variants of optical flow: Demons

- We have a single OF equation in 2 or 3 variables. I.e. many solutions possible



$$\nabla I \cdot \mathbf{u} = I - J$$

- Relates to the ‘aperture problem’
- One solution is

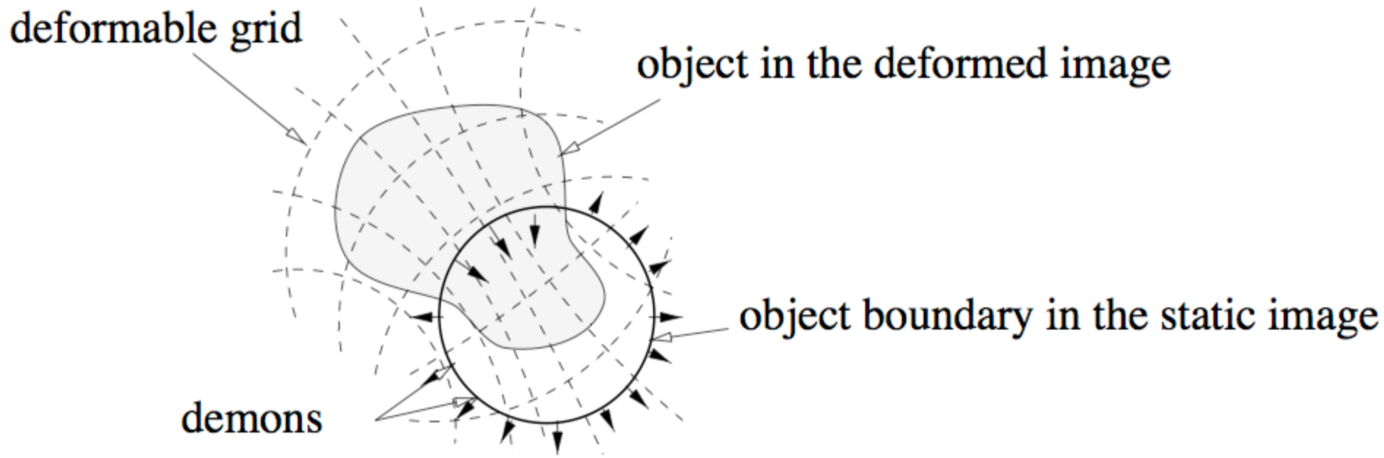
$$\mathbf{u} = \frac{(I - J)\ \nabla I}{|\,\nabla I\,|^2}$$

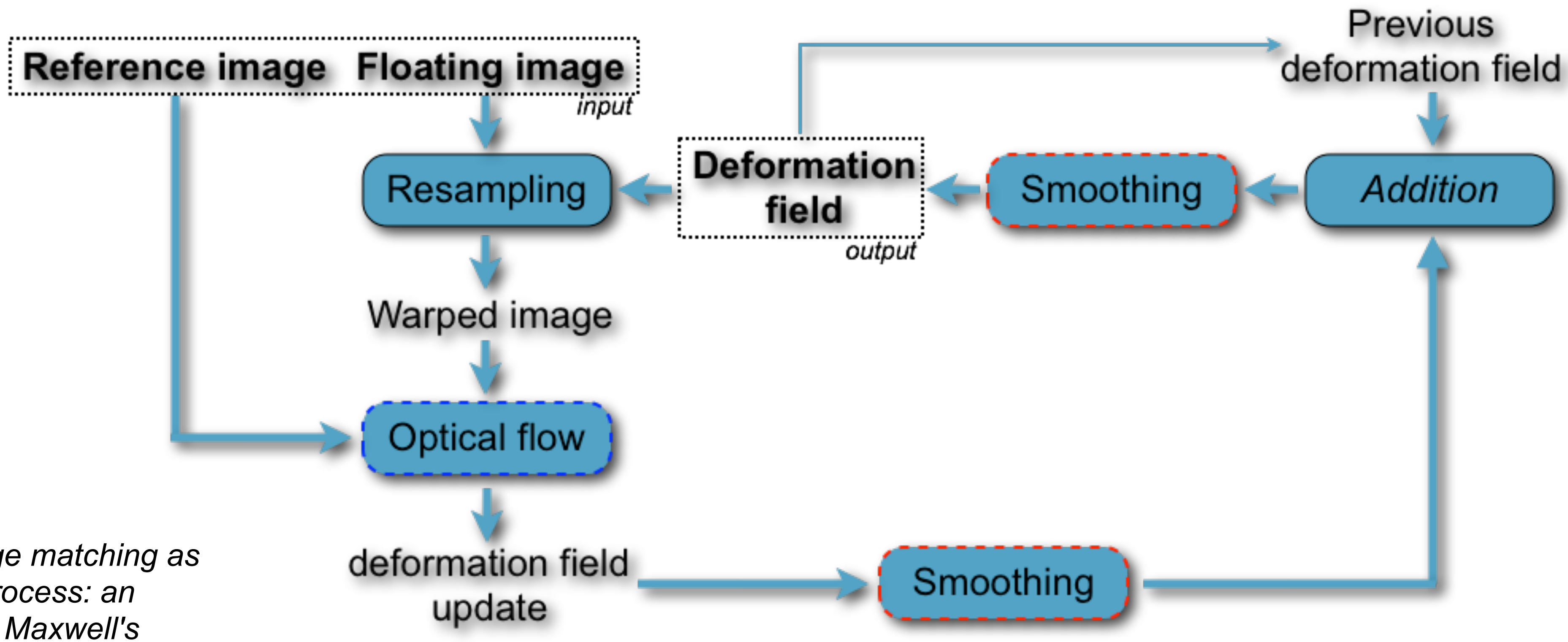
- To avoid instabilities this can be modified

$$\mathbf{u} = \frac{(I - J)\ \nabla I}{|\,\nabla I\,|^2 + (I - J)^2}$$

Thirion, *Image matching as a diffusion process: an analogy with Maxwell's demons*. Medical Image Analysis 2, 1998

Variants of optical flow: Demons





Thirion, *Image matching as a diffusion process: an analogy with Maxwell's demons*. Medical Image Analysis 2, 1998

Elastic registration

- Deformation modelled as a physical process: resembles stretching of an elastic material (e.g. rubber)
- Deformation governed by two forces
  - ***internal force***: caused by deformation of the elastic body, i.e. stress. Total internal force is zero if the body is in equilibrium.
  - ***external force***: acts on the elastic body and causes the body to deform away from the equilibrium.
- External force can ‘drive’ the registration based on
  - Distance between corresponding geometric features (points, lines or surfaces)
  - Voxel-similarity measure

Elastic registration

- Elastic deformation described by a linear partial differential equation (PDE):

$$\mu \nabla^2 \mathbf{u}(x,y,z) + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}(x,y,z)) + \mathbf{f}(x,y,z) = 0$$

$\mathbf{u}$	- Displacement field
$\mathbf{f}$	- External force
∇	- Gradient operator
∇^2	- Laplacian operator
λ, μ	- Lamé's elasticity constants (reflect material properties)

Elastic registration

- PDE can be solved using either
  - Finite differences and successive overrelaxation (SOR)
    - Produces a dense displacement field: Vector at every voxel
  - Finite element methods
    - Produces a sparse displacement field at the FEM nodes.
    - Displacement at every voxel can be obtained by interpolation

- Problems:
  - Cannot cope with large deformations
  - Cannot cope with changes in topology
- Extensions of elastic registration
  - Spatially varying elasticity parameters (Davatzikos, 1997)

Types of non-rigid transformations

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  - spline-based transformations
- Non-parametric transformations
  - optical flow type registration (e.g. Demons)
  - elastic registration
  - fluid or geodesic registration

Small deformation models

Large deformation models

Fluid registration

- Advantages:
  - Can cope with large deformations
  - Preserves topology

- Similar to elastic registration, uses a partial differential equation, but solves for **velocity field** $\mathbf{v}$

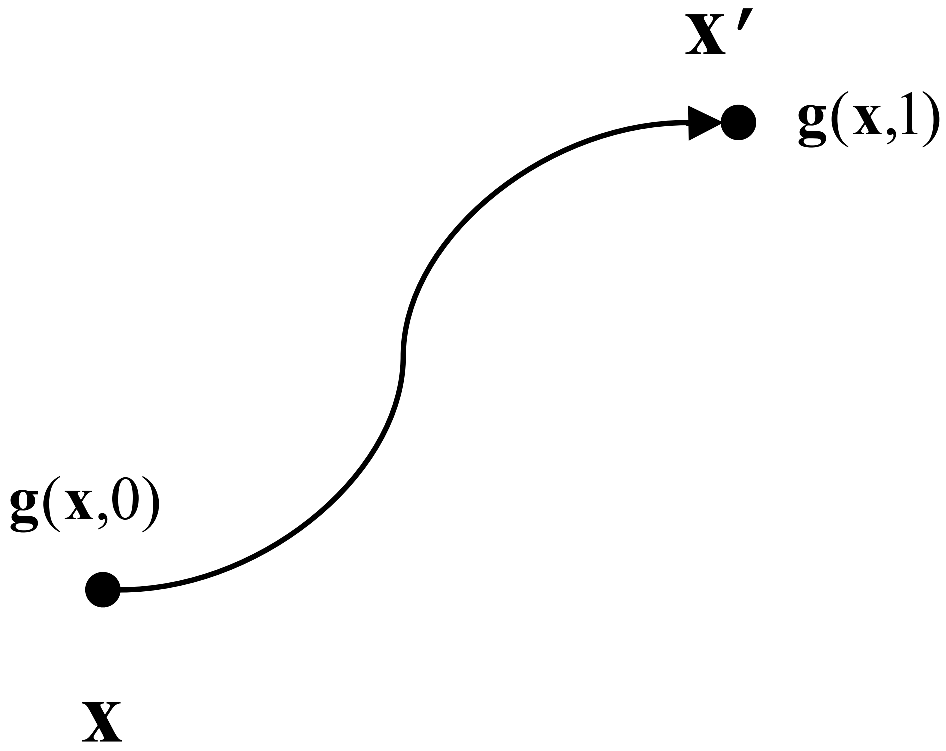
$$\mu\nabla^2\mathbf{v}(x,y,z) + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{v}(x,y,z)) + \mathbf{f}(x,y,z) = 0$$

Velocity is estimated over a series of time steps

Christensen, Deformable templates using large deformation kinematics, IEEE Transactions on Image Processing, 5 1996
Bro-Neilsen, Fast fluid registration of medical images, VBC 1131, 1996

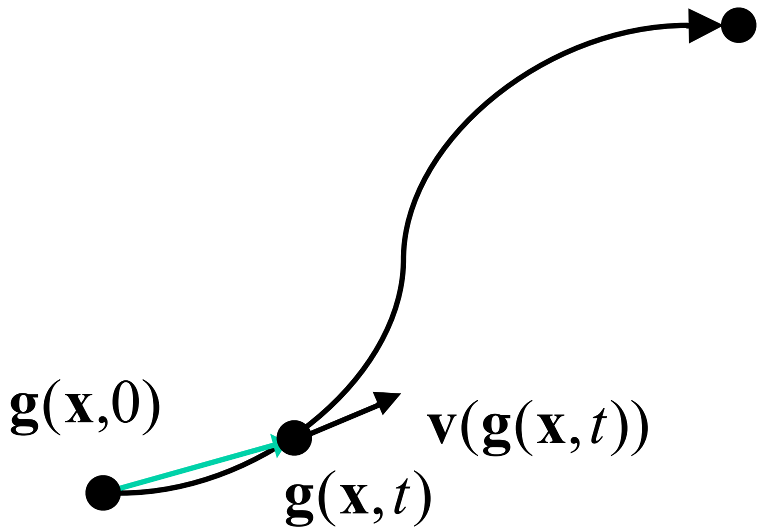
Fluid registration

- Deformation of point $\mathbf{x}$ is represented with a time varying displacement $\mathbf{g}(\mathbf{x},t)$
- $\mathbf{g}(\mathbf{x},t)$ traces out a path from $\mathbf{x}$ to its destination $\mathbf{x}'$
  - As t varies from 0 to 1



Fluid registration

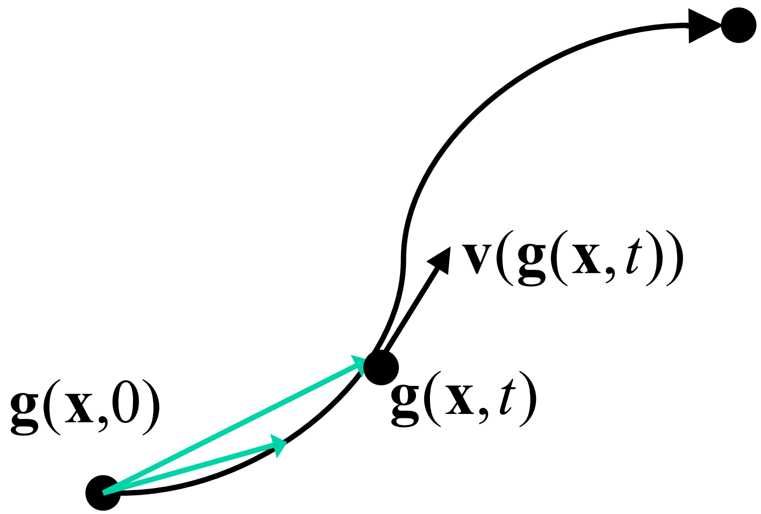
- Carried out in a series of steps
- At each step
  - Update current estimate of $\mathbf{v}$
  - Update $\mathbf{g}$ by adding $\mathbf{v}$ at current position



t = 0.00

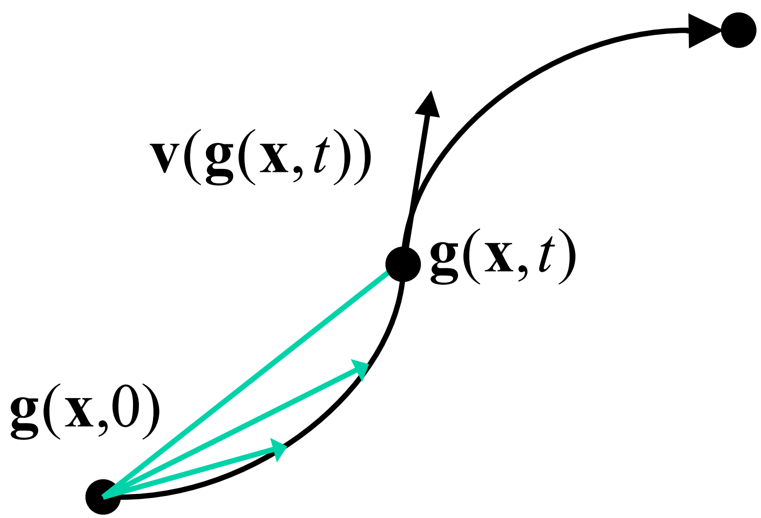
Fluid registration

- With each increment, we can update the moving image and recalculate the velocity field
- I.e. $\mathbf{v}$ varies in space and time



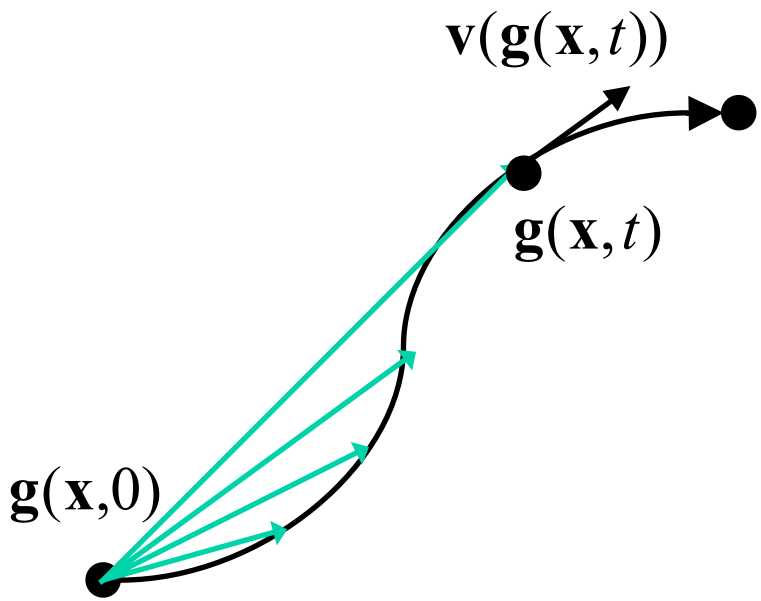
`t = 0.25`

Fluid registration



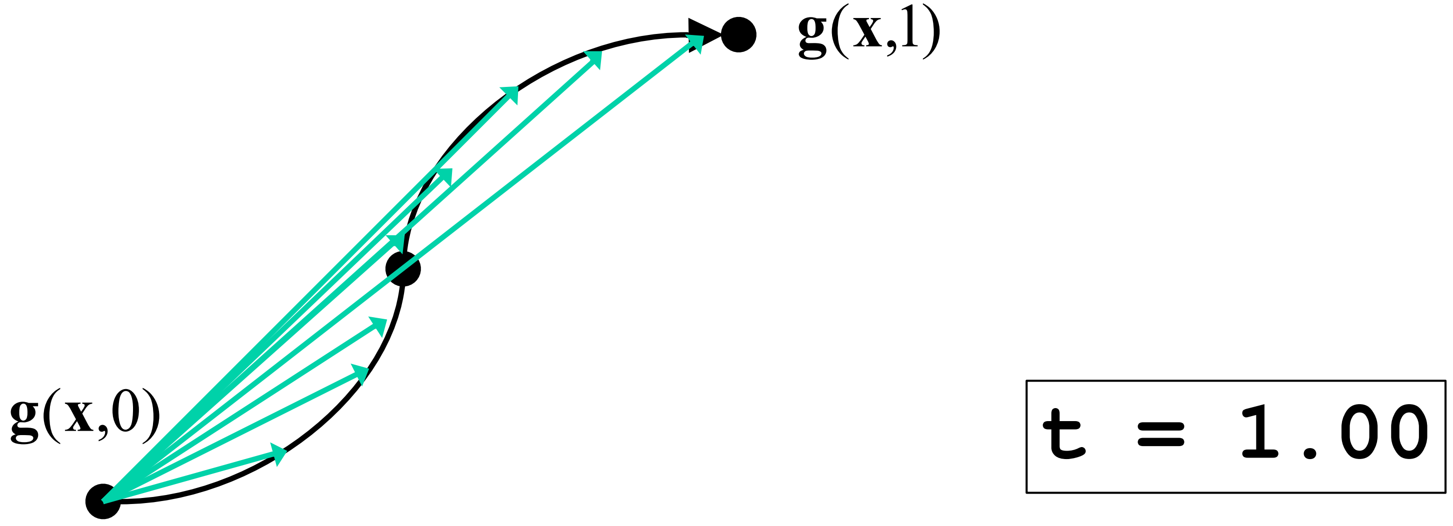
`t = 0.50`

Fluid registration



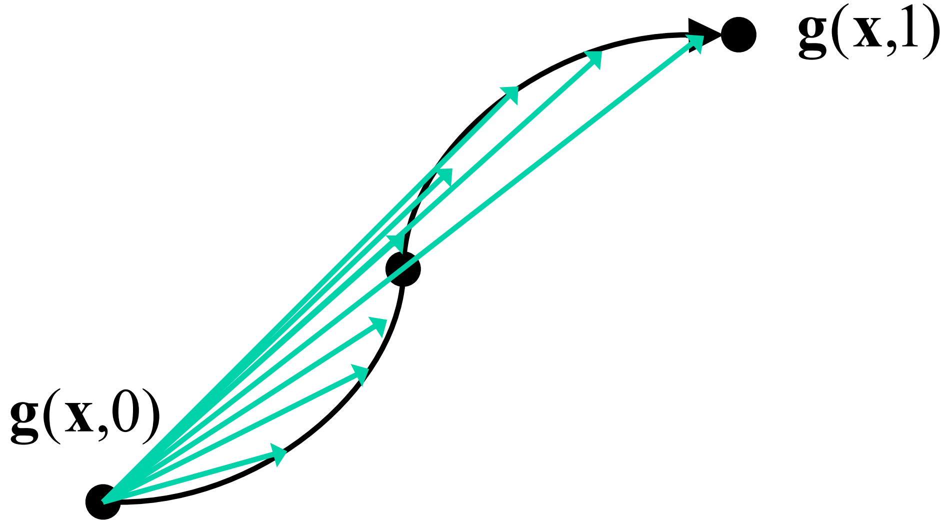
t = 0.75

Fluid registration



Fluid registration

$$\mathbf{x}' = g(\mathbf{x}, 1) = \mathbf{x} + \int_{t=0}^{1} \mathbf{v}(\mathbf{g}(\mathbf{x}, t))dt$$



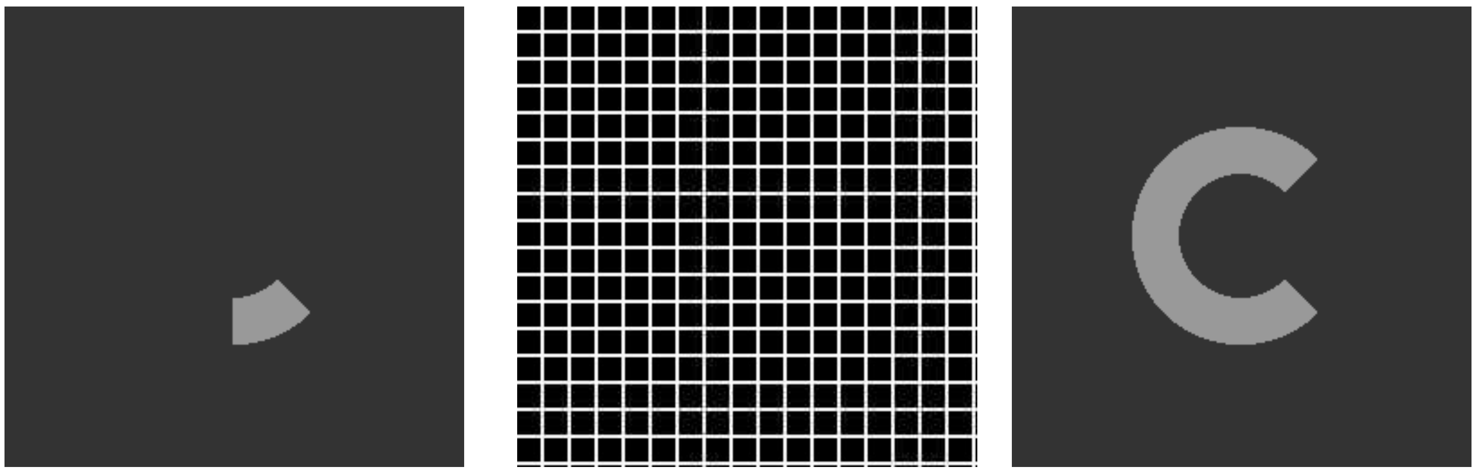
Geodesic registration

- Traditional fluid registration techniques
  - + Can deal with large deformations
  - + Are diffeomorphic
  - – Approximate a greedy solution for the deformation, not necessarily optimal
- Geodesic registration techniques
  - + Determine the shortest path (geodesic - in group of deformations)
  - + Resulting deformation can be used as a metric
- Several competing approaches:
  - – F. Beg et al. *Computing Large Deformation Metric Mappings via Geodesic Flows of Diffeomorphisms*. IJCV 2005.
  - – B. Avants et al. *Symmetric diffeomorphic image registration with cross-correlation*, Medical Image Analysis 2008

LDDMM registration

In fluid or geodesic registration transformations are constructed using the group of *diffeomorphisms* of the underlying coordinate system

Diffeomorphisms: one-to-one onto (invertible) and differential transformations. Preserve topology.



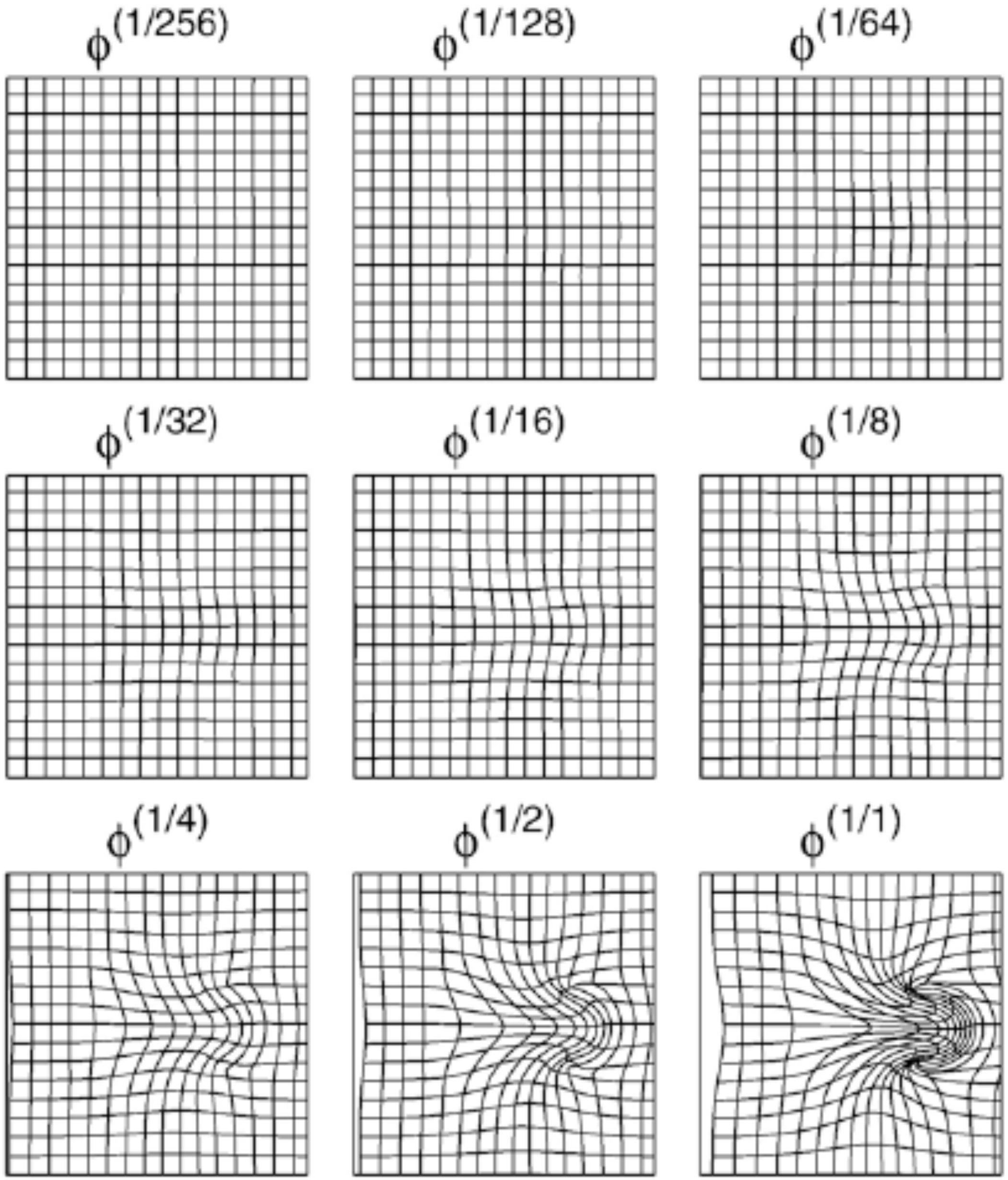
Thanks to S. Joshi

Alternatives to LDDMM registration

- Use <u>stationary</u> instead of time-varying velocity fields:
  - J. Ashburner, *A fast diffeomorphic image registration algorithm*, Neuroimage 2007
  - T. Vecauteren et al., *Diffeomorphic demons: Efficient non-parametric image registration*, NeuroImage 2009
- Integration of velocity fields can be solved very efficiently using scaling and squaring, e.g. for 3 steps we have:

$$\mathbf{g}(\mathbf{x}, \frac{1}{8}) \approx \mathbf{x} + \frac{\mathbf{v}(\mathbf{x})}{8}$$
$$\mathbf{g}(\mathbf{x}, \frac{1}{4}) = \mathbf{g}(\mathbf{x}, \frac{1}{8}) \circ \mathbf{g}(\mathbf{x}, \frac{1}{8})$$
$$\mathbf{g}(\mathbf{x}, \frac{1}{2}) = \mathbf{g}(\mathbf{x}, \frac{1}{4}) \circ \mathbf{g}(\mathbf{x}, \frac{1}{4})$$
$$\mathbf{g}(\mathbf{x}, 1) = \mathbf{g}(\mathbf{x}, \frac{1}{2}) \circ \mathbf{g}(\mathbf{x}, \frac{1}{2})$$

Scaling and squaring



Freely available implementations

SyN (ANTS based on ITK)

Symmetric registration
Non-stationary velocity field

Demons (part of ITK)

Gaussian regularisation
Various implementations

Dartel (part of SPM)

Stationary velocity field - scaling and squaring

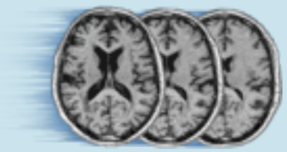
Non-parametric registration

IRTK

The Original Free-Form Deformation implementation

FNIRT (part of FSL)

Free-form deformation with multiple extra parameters



F3D (part of NiftyReg)

Free-form deformation with a focus on computation time

Elastix (based on ITK)

Free-form deformation using multiple filters

Parametric registration

Thanks !!!

Appendix on feature based methods

Following slides contain a few further details on registration using features.

Generic feature-based registration: Algorithm

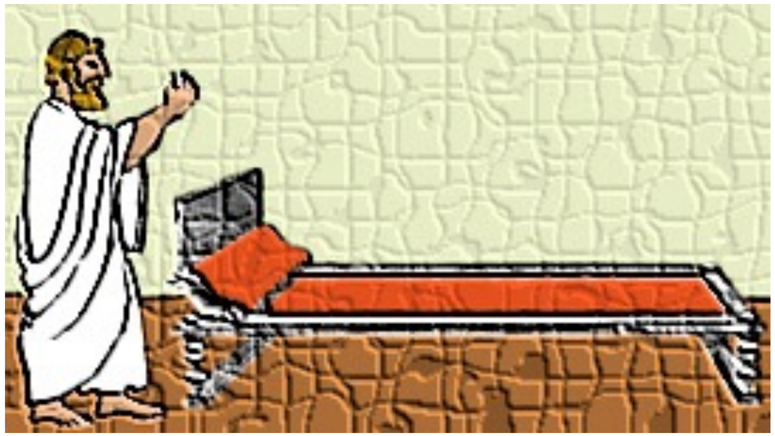
```
While dissimilarity > 0 and improvement possible
do
    Feature extraction
    Feature pairing
    Similarity formulation and outlier removal
    Dissimilarity reduction (optimization)
```

Great variety in how each step can be carried out depending on images and task!

Generic feature-based registration: *Using points*

- Orthogonal Procrustes problem:
  - Named after a robber in Greek mythology. Offered travellers opportunity to stay the night in a **perfectly** fitting bed.
  - Unfortunately, it was the guest who was altered to fit the bed, rather than the bed to fit the guest!
  - Short visitors were stretched to fit, and tall visitors had parts of their body cut off so that they would fit, with invariably fatal results.

The hero Theseus stopped this unpleasant practice by subjecting Procrustes to his own method.

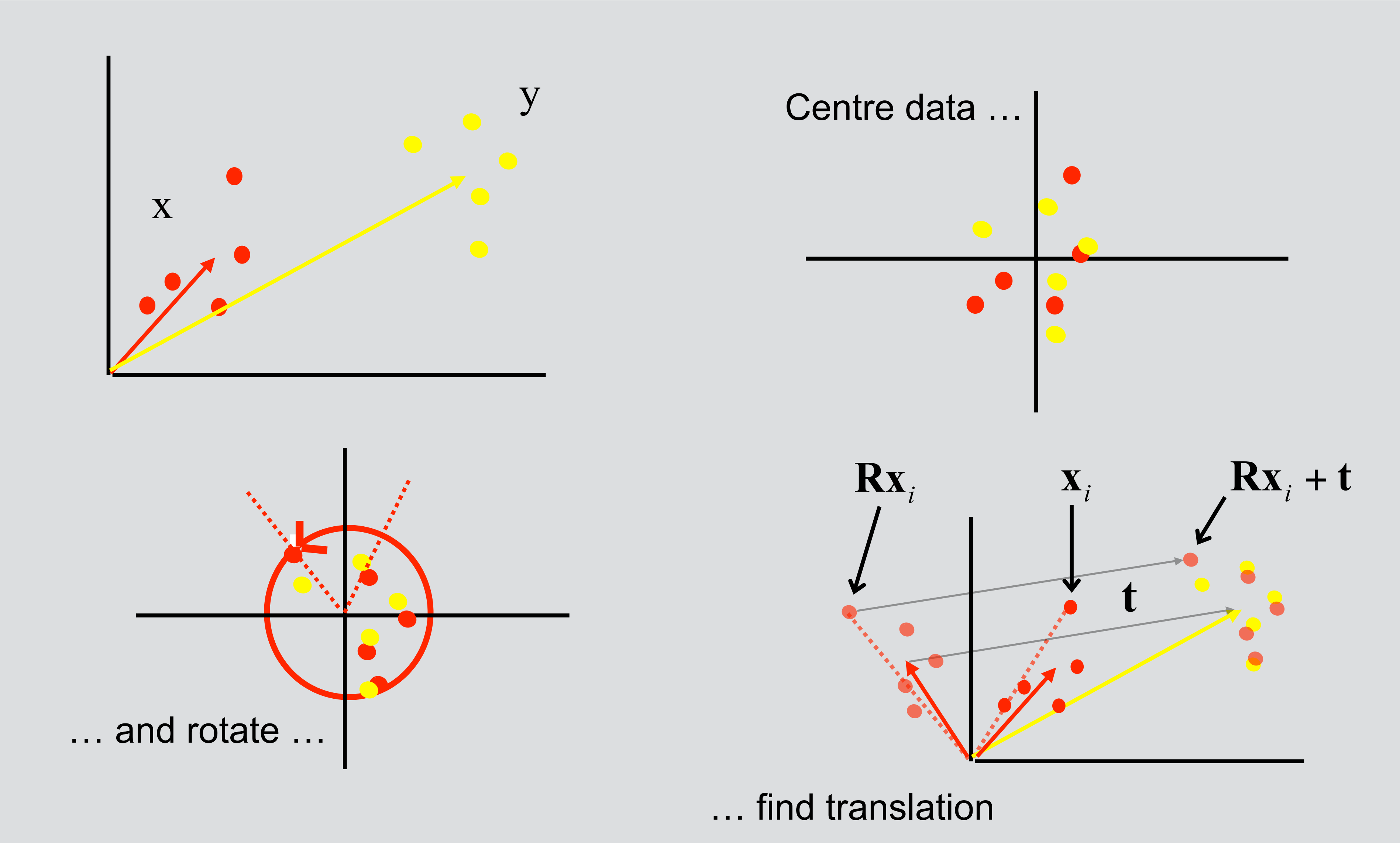


Generic feature-based registration: *Using points for rigid registration*

- Optimal fitting problem of least square type:
  - Closed-form solution for rigid (orthogonal) case
  - Given two sets of N points $\{\mathbf{x}_i\}$ and $\{\mathbf{y}_i\}$
  - with known correspondence
  - find the rigid-body transformation (rotation matrix $\mathbf{R}$ and translation vector $\mathbf{t}$) that minimizes the mean squared distance between the points:

$$\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{R}\mathbf{x}_i+\mathbf{t}-\mathbf{y}_i\|^2$$

Generic feature-based registration:
Using points for rigid registration



Generic feature-based registration: *Using points for rigid registration*

- Centre points:
  - Compute mean: $\bar{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i \qquad \bar{\mathbf{y}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{y}_i$
  - Centre wrt mean: $\tilde{\mathbf{x}}_i = \mathbf{x}_i - \bar{\mathbf{x}} \qquad \tilde{\mathbf{y}}_i = \mathbf{y}_i - \bar{\mathbf{y}}$
- Determine rotation matrix $\mathbf{R}$: singular value decomposition (SVD) of correlation matrix $\mathbf{H}$:

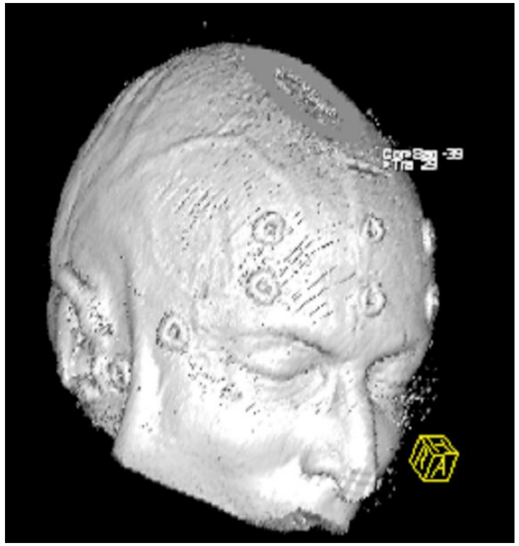
$$\mathbf{H} = \sum_{i=1}^{N}\tilde{\mathbf{x}}_i\tilde{\mathbf{y}}_i^t \qquad \mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T\,, \mathbf{U}^T\mathbf{U} = \mathbf{I}\,, \mathbf{V}^T\mathbf{V} = \mathbf{I}$$

$$\Lambda = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \qquad \mathbf{R} = \mathbf{V}\mathbf{D}\mathbf{U}^T\,, \quad \mathbf{D} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & d \end{pmatrix}\,, \quad d = \det(\mathbf{V}\mathbf{U}^T)$$

- Determine translation vector: $\mathbf{t} = \bar{\mathbf{y}} - \mathbf{R}\bar{\mathbf{x}}$

Generic feature-based registration:
Using points for rigid registration

- Anatomic landmarks
- Skin-affixed markers
- Bone-implanted markers
- Advantages of markers
  - *Fiducial** is independent of anatomy
  - Automatic algorithms for locating fiducial markers can take advantage of marker's shape and size in order to accurately and robustly compute the fiducial point



Wikipedia: A fiducial marker or fiducial is an object placed in the field of view of an imaging system which appears in the image produced, for use as a point of reference or a measure.

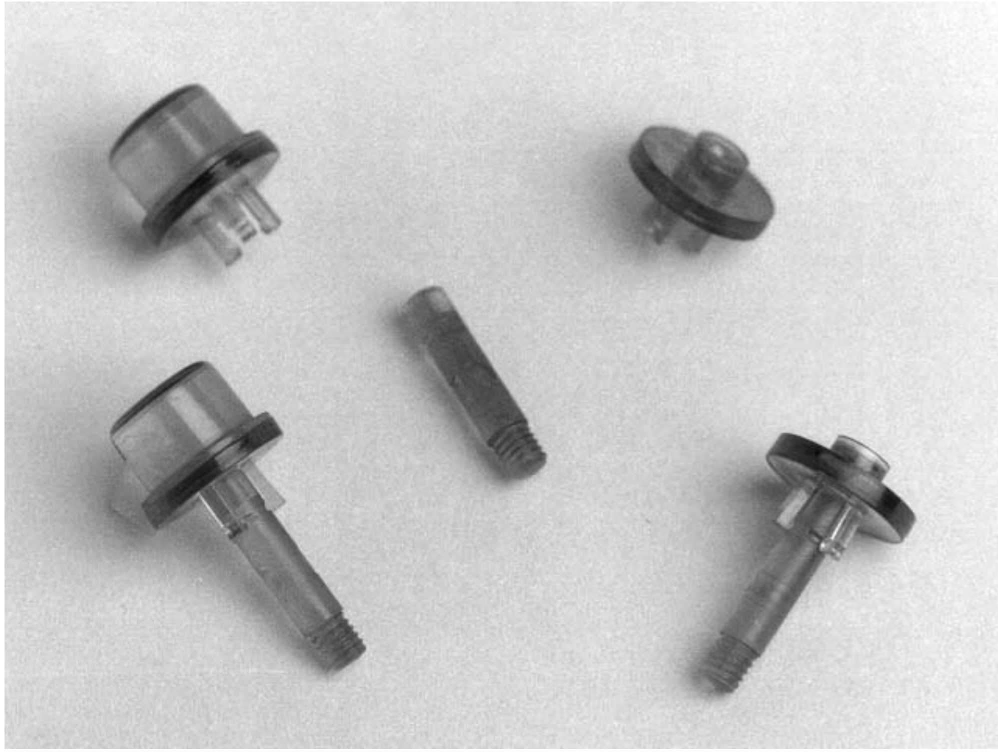
“Bummer of a birthmark, Hal!”

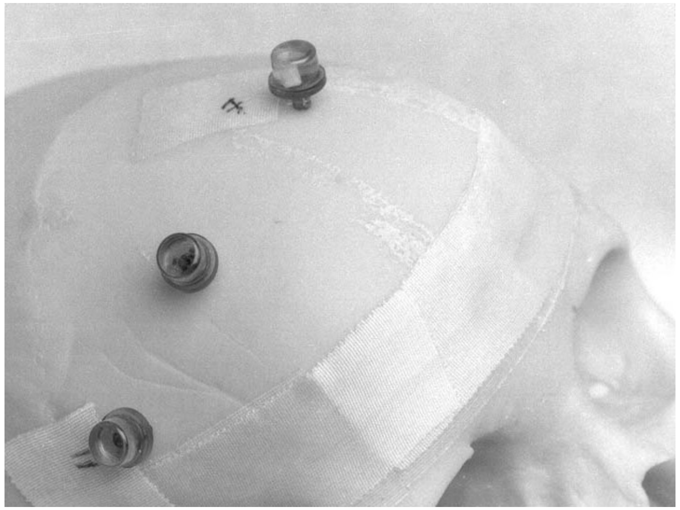
Skin-Affixed Markers

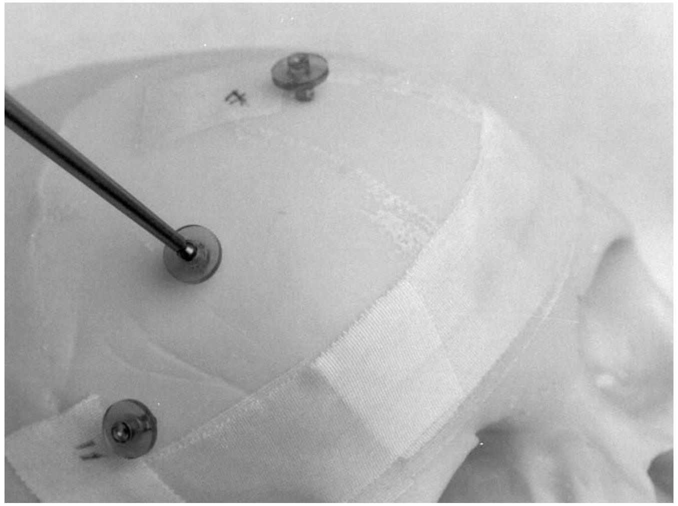
- Advantage: non-invasive
- Disadvantage: can move due to mobility of skin

Bone-Implanted Markers

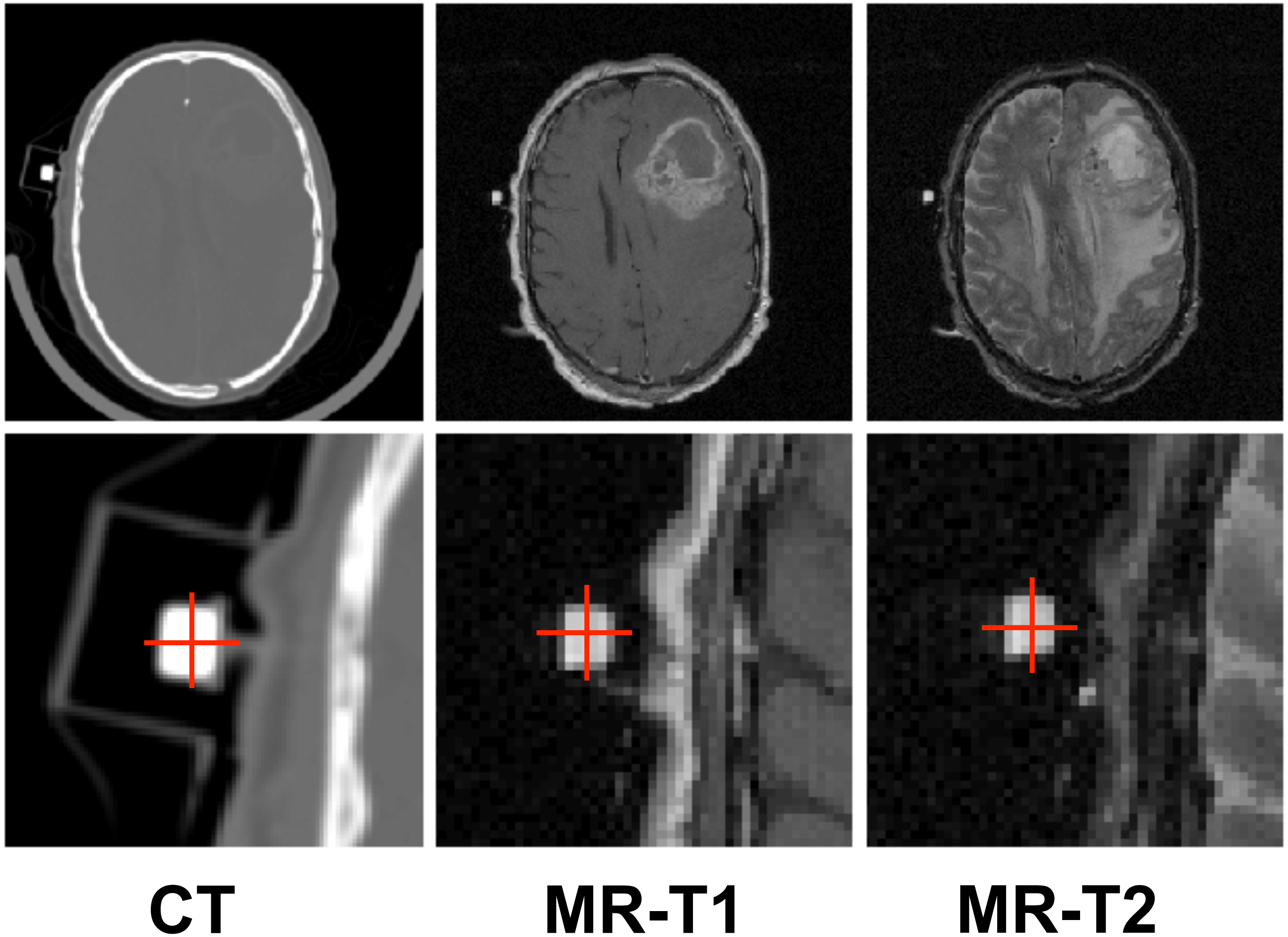
- Advantage: cannot move
- Disadvantage: invasive







Bone-Implanted Markers



Generic feature-based registration: *Using surfaces*

- Generally aligns a large number of points
- 3D correspondence of anatomy or pathology is often not known or unavailable
- The 3D boundary of an object is an intuitive and easily characterised geometrical feature that can be used for registration
- Surface-based methods involve determining corresponding surfaces in different images and/or physical space and finding transformation that best aligns these surfaces

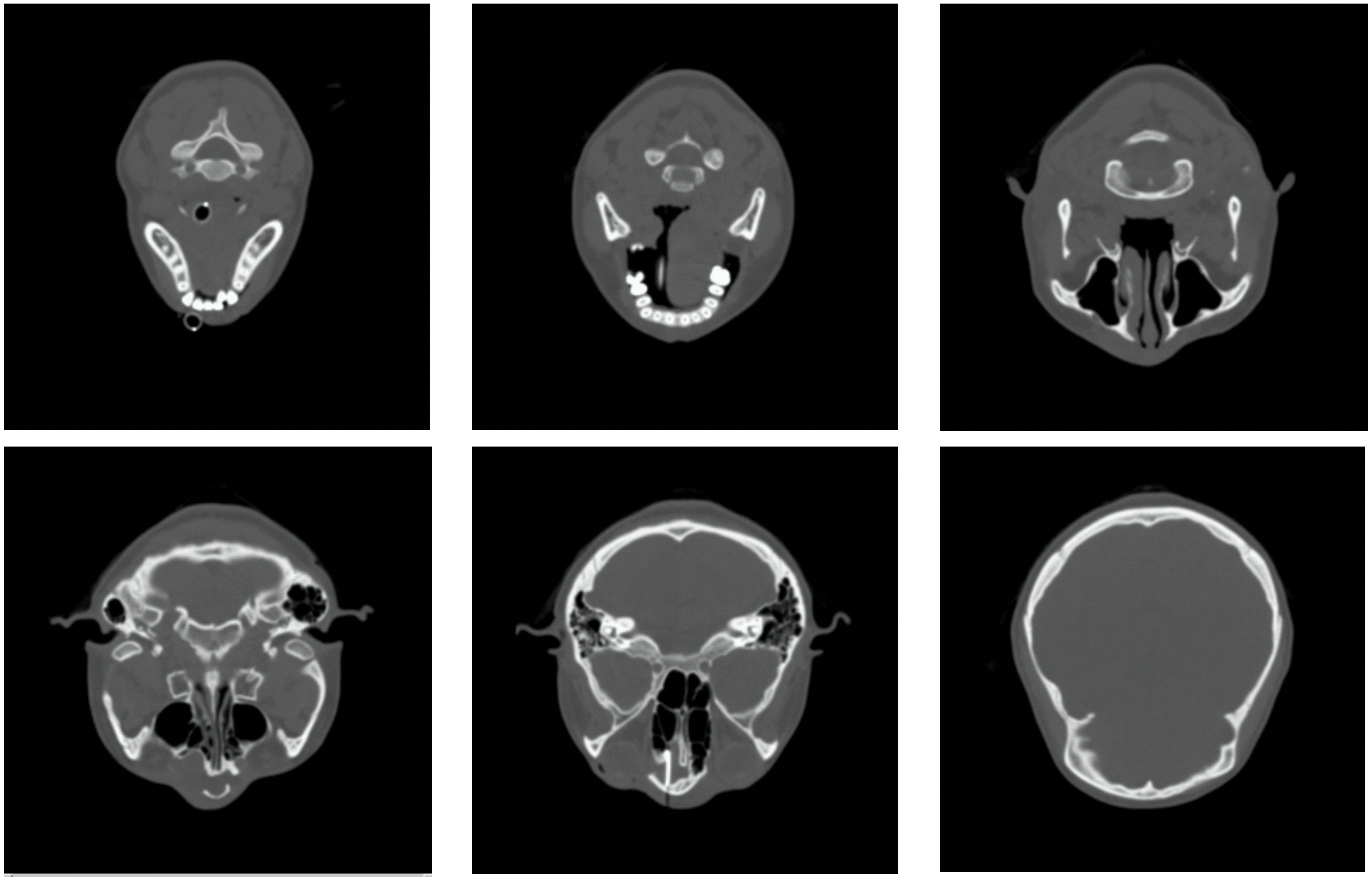
Generic feature-based registration: *Using surfaces*

- Skin surface (air-tissue interface)
- Bone surface (tissue-bone interface)
- Representations
  - Point sets (collection of points on the surface)
  - Faceted surfaces e.g., triangle set approximating surface
  - Implicit surfaces
  - Parametric surfaces, e.g., spline surface

Surface Extraction

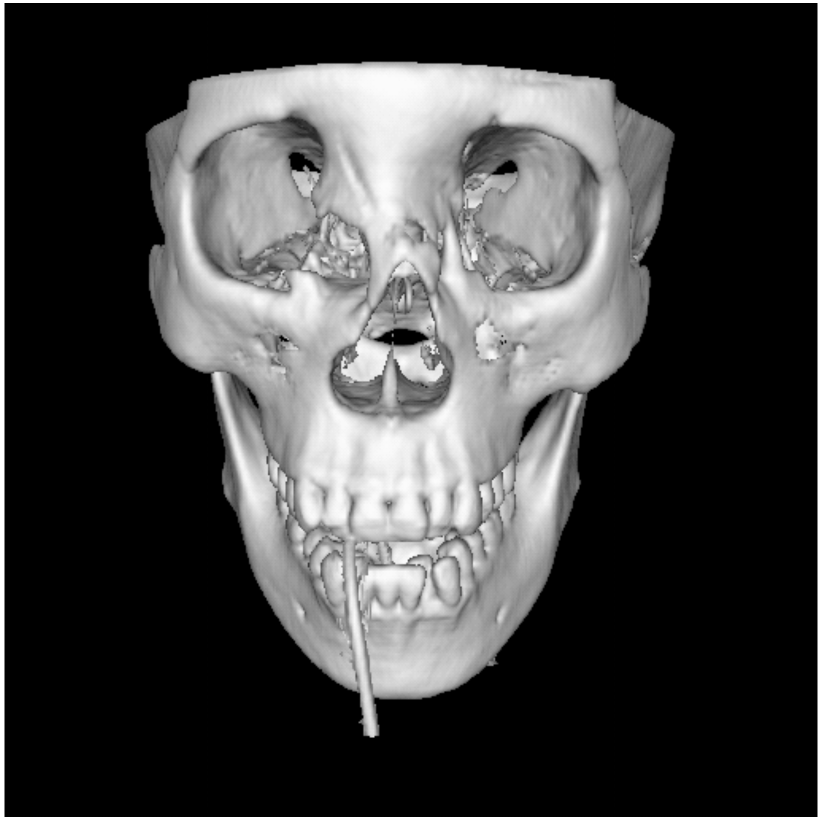
- Images
  - Isointensity contour extraction (Marching Cubes)
  - Deformable models
- Physical space
  - Laser range finders
  - Stereo video systems (photogrammetry)
  - Localizers
    - Articulated mechanical
    - Magnetic
    - Active and passive optical
  - Tracked ultrasound for bone surface

Marching Cubes: Example



Marching Cubes: Example

Skin surface



Bone surface

Generic feature-based registration:
Using surfaces

Given

- N surface points $\{\mathbf{x_i}\}$
- A surface $\mathbf{Y}$

Find transformation $\mathbf{T}$ that minimises the mean squared distance between the points and the surface:

$$D(\mathbf{T})\frac{1}{N}\sum_{i=1}^{N}\|\mathbf{T}(\mathbf{x}_i) - \mathbf{y}_i\|^2 \qquad \mathbf{y}_i = \underset{\mathbf{y}\in Y}{\operatorname{argmin}}\ \operatorname{dist}(\mathbf{x}_i, \mathbf{y})$$

$\mathbf{y}_i$ is the closest point on Y to $\mathbf{x}_i$, not necessarily corresponding

Generic feature-based registration: *Using surfaces*

- *Iterative Closest Point (ICP)* [Besl & McKay, PAMI 1992]
- To register *data* shape $\mathbf{X}$ to *model* shape $\mathbf{Y}$, decompose $\mathbf{X}$ into point set $\{\mathbf{x}_i\}$, then
  – Compute closest points $\{\mathbf{y}_i\}$ on $\mathbf{Y}$
  – Register points $\{\mathbf{x}_i\}$ to points $\{\mathbf{y}_i\}$
  – Apply resulting transformation to points $\{\mathbf{x}_i\}$
  – Repeat until convergence

Generic feature-based registration:
Using surfaces